Housing Price Dynamics: Predictability, Liquidity and Investor Returns

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December 2008

It is widely accepted that aggregate housing prices are predictable, but that excess returns to investors are precluded by the transactions costs of buying and selling property. We examine this issue using a unique data set -- all private condominium sales in Singapore during an eleven-year period. We model directly the price discovery process for individual dwellings. Our empirical results clearly reject a random walk in prices, supporting mean reversion in housing prices and diffusion of innovations over space. We find that, when house prices and aggregate returns are computed from models that erroneously assume a random walk and spatial independence, they are strongly autocorrelated. However, when they are calculated from the appropriate model, predictability in prices and in investment returns is completely absent. We show that this is due to the illiquid nature of housing transactions. We also conduct extensive simulations, over different time horizons and with different investment rules, testing whether better information on housing price dynamics leads to superior investment performance.

JEL Codes: E31, C23, R32
Key Words: Housing Market Liquidity, Price Discovery, Spatial Correlation

I. Introduction

The durability, fixity and heterogeneity of dwellings imply that transactions costs are significant in the housing market. In comparison to financial markets, and in comparison to the markets for most consumer goods, housing purchases require costly search to uncover the prices and attributes of commodities. Given the many frictions associated with the purchase of housing, it is hardly surprising that observed price behavior deviates from that predicted by simple models of economic markets. Inertia in the price adjustment process, either in aggregate prices (Case and Shiller, 1989, Hosios and Pesando, 1991) or in individual house prices (Englund, et. al., 1999, Hill, et. al., 1999), is widely reported. This is often regarded as evidence of housing market inefficiency (eg. Case and Shiller, 1990).

But in this geographical market, price signals exist in space as well as time. Many of the features which can lead to slow diffusion in the time domain may have analogous effects over space. Price information diffuses over space as well as time, and information costs alone can cause prices to deviate from random fluctuations.

This paper examines price discovery in a spatial market using a body of data almost uniquely suited to the analysis. We examine the prices of condominium dwellings in Singapore using all sales reported in the entire country during an eleven-year period. Multiple sales of the same condominium unit are observed, and all dwellings with market transactions are geocoded. We develop a model of price diffusion, and we incorporate a more general and more appropriate structure of the price discovery process at the level of the individual dwelling.

The literature on price discovery in housing markets is substantial. Following Case and Shiller (1989), others have documented predictable returns in housing markets by demonstrating that time series estimates of aggregate prices exhibit inertia in percentage changes (Guntermann
and Norrbom, 1991; Gatzlaff, 1994; and Malpezzi, 1999). We develop an explicit model of the spatial as well as temporal dependence of housing prices to evaluate the importance of these factors in affecting the course of individual housing prices. We compare the properties of aggregate housing price indexes and returns computed from our more general model with indexes computed from conventional models. We find that when aggregate investment returns are estimated from models which require that housing prices follow a random walk and that they be spatially independent, they are strongly predictable. However, when aggregate returns are estimated from more general models permitting mean revision and spatial correlation, predictability in aggregate investment returns is completely absent. We show that this arises from the illiquid nature of housing transactions and from persistent forecast errors in aggregate housing returns. The latter arises from inadequate treatment of correlation among the returns to housing investment over time and space.

We then analyze the economic implications of these statistical findings for investment in housing markets. In particular, we simulate the investment outcomes for an investor fully informed about spatial and temporal dynamics with the outcomes for an uninformed investor. Presumably, better information about housing market dynamics will lead to better investment performance in the housing market. We find that the investor with better knowledge of price diffusion over time and space outperforms the uninformed investor, capitalizing on this informational advantage. However, her superior performance appears to be bounded by relatively short holding periods and low transactions costs.

Section II develops a general model of housing prices that supports explicit tests for the spatial and temporal pattern of price movements. This section links our model to the widely employed method for measuring housing prices proposed more than forty years ago by Bailey,
Muth, and Nourse (1963), as well as its subsequent extensions (e.g., Case and Shiller, 1987). The data are described tersely in Section III. Our empirical results are presented in Sections IV, V, VI and VII. We test for random walks in space and time against the alternative of mean reversion, and we examine the link between pricing deviations at the individual level and aggregate price movements. We reconcile the puzzle of autocorrelated estimates of prices and returns. We also investigate investor behavior and housing market illiquidity in some detail. Section VIII is a brief conclusion.

II. A Micro Model of House Prices

The objects of exchange in the housing market are imperfect substitutes for one another. Indeed, dwellings with identical physical attributes may differ in market price simply because the price incorporates a complex set of site-specific amenities and access costs. But few dwellings have identical physical characteristics; thus comparison-shopping is more difficult and more expensive than in most other markets.

Moreover, housing transactions are made only infrequently, so households must consciously invest in information to participate in this market. As a result, the market is characterized by a costly matching process. Market agents, buyers, and sellers are heterogeneous, and they differ in information and motivation; commodities are themselves heterogeneous. Consequently an observed transaction price for a specific unit may deviate from the price ordained in a simpler environment.

Buyers, sellers, appraisers, and real estate agents estimate the “market price” of a dwelling by utilizing the information embodied in the set of previously sold dwellings. The
usefulness of these sales as a reference depends upon their similarity across several dimensions: physical, spatial, and temporal. Inferences about the “market price” of the dwelling can be drawn only imperfectly from a set of past sales, because dwellings differ structurally, enjoy different locational attributes, and are valued under different market conditions by different actors over time. Because dwellings trade infrequently, the arrival of new information about market values is slow. From an informational standpoint, the closest comparable sale across these various dimensions may be the last sale of the same dwelling. Alternatively, the most comparable sale may be the contemporaneous selling price of another dwelling in close physical proximity.

An attempt to uncover the market value of a dwelling is further complicated by the fact that a sales price is not only a function of observable physical characteristics, but also of unobserved buyer and seller characteristics such as their urgency to conclude a transaction (Quan and Quigley, 1991). For any given sale, all that is known is that an offer was made by a specific buyer that was higher than a specific seller’s reservation price.

We develop a model with spatially and temporally correlated errors in a repeat sales framework. Innovation processes over time are assumed to be continuous, but sales occur sporadically. At any point in time, the prices of houses are dependent over space. In the determination of the price of a house, the weights attributable to neighboring houses depend upon their proximity to the house. But the prices of neighboring houses are also observed only infrequently.

Let the log sale price of dwelling $i$ at time $t$ be

$$V_{it} = P_t + Q_{it} + e_{it} = P_t + X_{it} \beta + e_{it},$$

(1)
where $V_{it}$ is the log of the observed sales price of dwelling $i$ at $t$, and $P_t$ is the log of aggregate housing prices. $Q_{it}$ is the log of housing quality, and can be parameterized by $X_{it}$, the set of housing attributes and by a set of coefficients, $\beta$, which price those attributes. If a sale is observed at two points in time, $t$ and $\tau$, and if the quality of the dwelling remains constant during the interval, then

$$(2) \quad V_{it} - V_{it\tau} = P_t - P_\tau + (X_{it} - X_{it\tau}) \beta + e_{it} - e_{it\tau}$$

$$= P_t - P_\tau + e_{it} - e_{it\tau}.$$  

With constant quality, (2) identifies price change in the market. Equation (2) also shows that return on an individual dwelling can be decomposed into an aggregate return $(P_t - P_\tau)$ and an idiosyncratic return $(e_{it} - e_{it\tau})$.

Let the idiosyncratic part of the house price (or the error term), $e_{it}$, consist of two components that are realized for each individual dwelling at the time of sale: $\eta_{it}$, an idiosyncratic innovation without persistence; and $\varepsilon_{it}$, an idiosyncratic innovation with persistence, $\varepsilon_{it} = \lambda \varepsilon_{it-1} + \mu_{it}$. In addition, assume that the value of any particular dwelling depends also on innovations that occur to other dwellings contemporaneously. We assume this spatial correlation depends on the distance between units.

$$(3) \quad e_{it} = \rho \sum_{j=1}^{N} w_{ij} e_{jt} + \xi_{it} = \rho \sum_{j=1}^{N} w_{ij} e_{jt} + \varepsilon_{it} + \eta_{it} = \rho \sum_{j=1}^{N} w_{ij} e_{jt} + \lambda \varepsilon_{it-1} + \eta_{it} + \mu_{it},$$

where $w_{ij}$ is some function of the distance between unit $i$ and $j$ and $N$ is the number of dwellings in the economy. Let $E(\eta_{it} \eta_{jt}) = 0$ and $E(\varepsilon_{it} \varepsilon_{jt}) = 0$, $E(\eta_{it}^2) = \sigma_\eta^2$ and $E(\mu_{it}^2) = \sigma_\mu^2$. The value of a particular dwelling depends, not only on its own past and contemporaneous innovations, but also on innovations of other dwellings, past and contemporaneous.
In vector notation, expression (3) is

\[ \mathbf{e}_t = \rho \mathbf{W} \mathbf{e}_t + \xi_t, \]

where \( \mathbf{e}_t \) is a vector of \( e_{it} \) for all the dwellings at time \( t \), \( \mathbf{W} \) is a weight matrix, some measure of the distance between dwellings, and \( \xi_t \) a vector of \( \xi_{it} = \lambda t e_{i,t-1} + \eta_{it} + \mu_{it}, \) for all dwellings. By solving for \( \mathbf{e}_t \) and taking the difference between two sales at times \( t \) and \( s \), we have

\[ \mathbf{e}_t - \mathbf{e}_s = (I - \rho \mathbf{W})^{-1} (\xi_t - \xi_s). \]

The variance-covariance matrix of (5) is

\[ \mathbb{E} \left[ (\mathbf{e}_t - \mathbf{e}_s)(\mathbf{e}_t - \mathbf{e}_s)^\prime \right] = (I - \rho \mathbf{W})^{-1} \mathbb{E} \left[ (\xi_t - \xi_s)(\xi_t - \xi_s)^\prime \right] (I - \rho \mathbf{W})^{-1}. \]

Equations (5) and (6) indicate that when the prices of dwellings are autocorrelated over time and space, the price of any unit in the market at any period will be predictably related to those of other units at other periods.

Transactions on dwellings occur only infrequently. Consider the covariance in errors between a dwelling \( i \) sold at \( t \) and \( s \) and another dwelling \( k \) sold at \( \tau \) and \( \varsigma \), \[ \mathbb{E}[e_{it} - e_{is})(e_{k\tau} - e_{k\varsigma})]. \] Let \( \Psi = \mathbb{E}[ (\xi_t - \xi_s)(\xi_t - \xi_s)^\prime] \) and \( \Pi = (I - \rho \mathbf{W})^{-1} \). Thus,

\[ \mathbb{E}\left[ (\mathbf{e}_{it} - \mathbf{e}_{is})(\mathbf{e}_{k\tau} - \mathbf{e}_{k\varsigma})^\prime \right] = \Pi \Psi \Pi = \begin{bmatrix} \pi_1' \\ \pi_2' \\ \vdots \\ \pi_N' \end{bmatrix} \begin{bmatrix} \Psi_1 & \Psi_2 & \cdots & \Psi_N \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ \vdots \\ \pi_N \end{bmatrix} \]

The elements of this expression are,

\[ \mathbb{E}[e_{it} - e_{is})(e_{k\tau} - e_{k\varsigma})] = \pi_i \Psi_{ik}. \]

Now consider an element of the covariance matrix, \( \Psi \). Note that
(9) \[ E(\xi_i, \xi_j) = \mathcal{A}^{j-i} \left( \frac{\sigma_2^2}{1-\lambda^2} \right) + I(t = \tau)\sigma_n^2, \text{ if } i = j, \]
\[ = 0, \text{ otherwise,} \]
where \( I(\cdot) \) is an indicator function. For sales of a given dwelling at time \( t, s, \tau \) and \( \xi \),
\[ E(\xi_i - \xi_{is}, (\xi_{i\tau} - \xi_{is})) = \mathcal{A}^{j-i} \left( \mathcal{A}^{j-i} - \mathcal{A}^{j-i} + \mathcal{A}^{j-i} \right) \left( \frac{\sigma_2^2}{1-\lambda^2} \right) + \left[ I(t = \tau) - I(t = \xi) - I(s = \tau) + I(s = \xi) \right] \sigma_n^2. \]

Therefore, the variance-covariance matrix is
\[ \Psi = E(\xi_i - \xi_{is}, (\xi_{i\tau} - \xi_{is})) = E(\xi_i - \xi_{is})(\xi_{i\tau} - \xi_{is}) = \sigma_n^2 \times I. \]

Finally, the variance-covariance matrix of innovations between a dwelling \( i \) sold at \( t \) and \( s \) and another dwelling \( k \) sold at \( \tau \) and \( \zeta \) is
\[ E(\epsilon_{it} - \epsilon_{is}) (\epsilon_{k\tau} - \epsilon_{k\zeta}) = \pi_i \Psi \pi_k = \pi_i \{ E(\xi_i - \xi_{is})(\xi_{i\tau} - \xi_{i\zeta}) \} \times \pi_k \]
\[ = \left( \mathcal{A}^{j-i} - \mathcal{A}^{j-i} + \mathcal{A}^{j-i} \right) \left( \frac{\sigma_2^2}{1-\lambda^2} \right) + \left[ I(t = \tau) - I(t = \xi) - I(s = \tau) + I(s = \xi) \right] \sigma_n^2 \times \pi_i \pi_k. \]

Equation (12) indicates how the variance-covariance matrix of residuals from the regression specified in (2) can be used to identify the temporal and spatial components of house price persistence, \( \lambda \) and \( \rho \), respectively. Identification requires observing at least two transactions for each dwelling and observing the distance of each dwelling from all others in the market.

Note that this model of housing prices specializes to that of Bailey, Muth and Nourse (1963) when \( \lambda = \rho = 0 \), to that of Case and Schiller (1987) when \( \lambda = 1, \rho = 0 \) and to those of
Hill, et. al., (1997) and Englund, et. al., (1999) when $\rho = 0$. When, $\rho = 0$ so that no spatial correlation is present, the variance of the return on an individual dwelling between $t$ and $s$ is

$$\text{var}(V_{it} - V_{is}) = \frac{2\sigma^2_{\mu}}{1 - \lambda^2} (1 - \lambda^{t-s}) + 2\sigma^2_{\eta},$$

which is concave in the transaction interval.

In Case and Shiller’s (1989) model, with $\lambda = 1$ and $\rho = 0$, the error term in individual housing price follows $\eta_{it} = \mu_{it} + \eta_{it}$ where $\mu_{it} = \epsilon_{i,t-1} + \mu^t_{it}$. Then, the variance of the return on an individual dwelling is

$$\text{var}(V_{it} - V_{is}) = (t-s)\sigma^2_{\mu} + 2\sigma^2_{\eta}.$$

The variance increases linearly with the length of the time interval between transactions. Thus, with mean reversion in the data, a model based on a random walk assumption underestimates the return variances for housing transactions over short intervals, but overestimates the variances for housing transactions over long intervals.¹ The housing price indexes published by U.S. government agencies (e.g., the OFHEO price indices for metropolitan areas) are based upon the repeat sales model developed above with $\lambda=1$ and $\rho=0$. But the computation procedures do include a second order term in the variance estimation (See Abraham and Schauman, 1991, Calhoun, 1996), so that the variance increases at a diminishing rate with the time interval between sales.

$$\text{var}(v_{it} - v_{is}) = (t-s)\sigma^2_{\mu} + (t-s)^2\sigma^2_{\mu} + 2\sigma^2_{\eta}.$$

¹ This is because the return variance is concave when the dwelling price follows a spatio-temporal correlation process (or a mean reverting process).
III. Data

The analysis below is based upon all private condominium sales in Singapore during an eleven-year period. Non-landed properties (apartments and condominiums) account for roughly two-thirds of the Singapore housing stock, and units in condominiums account for almost forty percent of private residential housing in land-scarce Singapore.²

The data include all transactions involving condominium dwellings during the period from January 1, 1990 to December 31, 2000.³ An extensive set of physical characteristics of the dwellings is recorded. The date of the sale is recorded as well as the date of occupancy. In addition, the address, including the postal code, is reported. The postal code identifies the physical location – the block of flats or, quite often, the specific building. A matrix of distances among Singapore’s fifteen hundred postal codes permits each dwelling to be located spatially. The data set includes transactions among dwellings in the standing stock, sales of newly constructed dwellings, and presales of dwellings under construction (where sales may be consummated several months before the date construction is actually completed).

The panel nature of the data permits us to distinguish dwellings sold more than once, and this identifies the models specified in Section II. By confining the sample to dwellings in multifamily properties, we eliminate the types of dwellings for which additions and major renovations are feasible. The sample of multifamily dwellings is thus less likely to include those for which the assumption of constant quality between sales (see equation 2) is seriously violated.

Singapore data offer another advantage in estimating the model of housing prices, namely a spatial homogeneity of local public services (e.g., police protection, neighborhood schools), especially when compared to cities of comparable size in North America. During the

² See Sing (2001) for an extensive discussion of the condominium market in Singapore.
decade of the 1990s, there was no discernible trend in the quality of neighborhood attributes of the bundle of housing services.  

Table 1 presents a summary of the repeat sales data used in the empirical analysis reported below. There are several points worth noting. First, confirming the infrequency of housing transactions, the number of dwellings sold more than once is less than twenty percent of the population of dwellings sold during the eleven-year period. Only three percent of the 52,337 dwellings were sold more than twice in the eleven-year period.

Second, the average selling prices tend to be higher for dwellings sold more frequently. The rate of appreciation is also higher. On average, dwellings sold five times appreciate almost twice as fast as dwellings sold only twice. For the dwellings sold more frequently, price appreciation tends to be more volatile. Transactions involving high-turnover dwellings are apparently riskier, but this risk is compensated by higher returns.

Third, the intervals between sales are longer for dwellings sold infrequently. In part, this is an artifact of the fixed sampling framework. For presold dwellings, the average elapsed time between sale and completion of construction is largest for those sold least frequently. This is inconsistent with the popular belief in Singapore that presales are associated with “speculation” in the housing market.

Fourth, there are some differences in the characteristics of the dwellings sold more frequently. They tend to be larger in area, contain more rooms, and they are more centrally located. Their transit access is similar to that of dwellings sold less frequently.

3 The data have been supplied by the Singapore Institute of Surveyors and Valuers (SISV) which gathers transactions data from a variety of sources including legal registration records and developers’ sales records.
4 One possible exception to this may be accessibility, where improvement in the transport system and its pricing may have altered the workplace access of certain neighborhoods.
The data on condominium sales supports a price index regression model of the form of equation (2),

$$(15) \quad V_{it} - V_{is} = P_j D_{ij} - P_j D_{is} + \gamma \kappa_{it} - \gamma \kappa_{is} + e_{it} - e_{is},$$

where $D_{ij}$ is a variable with a value of 1 for the month $j$ in which condominium $i$ is sold and zero in other months, and $P_j$ is the estimated coefficient for this variable. There are 132 of these time variables, one for each month between 1990 and 2000. If dwelling $i$ has been presold, $\kappa_{it}$ is the time interval between the transaction date and the completion of construction. For dwellings sold after completion of construction, $\kappa_{it}$ is zero. Thus, the estimated coefficient $\gamma$ measures the monthly discount rate for presold dwellings, i.e., the discount for unrealized service flows from dwellings which have been purchased but which are not yet available for occupancy. The purchase of a dwelling before completion, or even before construction, is not unique to Singapore and has become rather common in condominium sales, for example in vacation properties in the U.S. Pre-sale contracts provide liquidity to developers and insurance to consumers against unanticipated price increases in the market. Of the 11,883 pairs of transactions noted in Table 1, 305 consist of presale pairs. For another 5,204 pairs, the first sale was made sometime before the property was completed.

IV. The Diffusion of House Price Innovations

We assume the error terms in equation (3), $\eta_{it}$ and $\mu_{it}$, are normally distributed. The log likelihood function for the observed sample of condominium sales is thus

$$(16) \quad \log L(P, \gamma, \lambda, \rho, \sigma_{\eta}^2, \sigma_{\mu}^2) = -\frac{T}{2} \log(2\pi) + \frac{1}{2} \left( \log \left| \Sigma \right| \right) - \left( \delta \Sigma^{-1} \delta \right),$$
where \( \Sigma = [\pi_i^T \Psi \pi_i^T] \) and \( \delta = [V_a - V_{ia} - P_PD_a - P_D + \gamma k_a - \gamma k_{ia}] \). We estimate the parameters, \( \lambda, \rho, \gamma, P, \sigma_{\eta} \) and \( \sigma_{\mu} \), by maximizing the log likelihood (16), based on 11,883 observations of repeat sales of 10,288 dwellings sold two or more times. In (3), the weights are assumed to be inversely related to distance, up to 250 meters.\(^6\) The influence of any transaction extends for roughly 200,000 square meters in the surrounding area.

Table 2 reports the estimated error structure when it is assumed that the price of an individual dwelling follows a spatio-temporal correlation process, a mean reverting process, and a random walk process, respectively. In the most general model, Column A, the estimated serial correlation coefficient, \( \rho \), implies a large persistence in individual housing prices, with a half life of more than six months. The estimated spatial correlation coefficient, 0.55, implies a slow spatial diffusion. These coefficients are quite precisely estimated; the estimated value for \( \rho \), 0.89, is significantly different from one by a wide margin. The estimated coefficient for \( \gamma \), the discount for the period between sale and dwelling completion (for presold units), is 14 basis points. This represents a 1.7 percent annual discount for a dwelling unit sold today for occupancy a year hence. The magnitude of the discount is not trivial. During this period aggregate housing prices rose, on average, by 0.4 percent monthly; thus, the discount for presold units reduced the net price appreciation for consumers by one third.

The second column reports parameter estimates for the model when individual house prices are allowed to follow a mean reverting process, but with \( \rho \) assumed to be zero. The estimated serial correlation coefficient, \( \lambda \), is 0.72, somewhat smaller than the estimate in Column 5.

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\(^5\) Presales are widely employed in China, for example, to finance development of new housing estates. See Deng and Lui (2008).

\(^6\) This expedites computation of (16) by making the \( \Sigma \) matrix sparse. When \( \rho \neq 0 \), the \( \Sigma \) matrix is a full square matrix with a length equal to the number of observations. However, with some cutoff, the \( W \) matrix is sparse. Since \( \pi_i \) is the \( i \)-th row of \( (I - \rho W)^{-1} \), when \( W \) is sparse, most of the elements of \( \pi_i \) will be zero. This reduces
A. Likelihood ratio tests reject a random walk in house prices ($\lambda = 1$) and serially uncorrelated house prices ($\lambda = 0$) by wide margins, $\chi^2 = 16,564$, and $\chi^2 = 1,670.6$ respectively. The estimated value of $\lambda$ suggests that the half-life of a one-unit shock to housing prices is about two months.

The third column reports parameter estimates for a model in which individual house prices are assumed to follow a random walk process without spatial autocorrelation. Following OFHEO, the table reports the model with a quadratic term, as in (14’), which also fits the data better. These results suggest that the variance in house prices increases with transaction intervals up to 37 months.

Figure 1 presents estimated monthly price indexes derived from the three models reported in Table 2. The estimated price index from the model with a mean reverting process and the index from the model with a spatio-temporal correlation process appear to move quite closely. The estimated price index from the random walk model is consistently lower than that implied by the other two models.

V. The Course of Housing Prices, Investment Returns and their Predictability

A. Predictability of Housing Returns

Although Figure 1 reports similar patterns for the course of housing prices of Singapore dwellings, the investment returns implied by these aggregate indices are quite different. Ignoring transactions costs and leverage, the return in any period, $R_t$, is the change in the asset value plus the dividend (i.e., the rental stream, $S_t$, enjoyed during the period).

computation time considerably. We experimented with several assumed cutoff values. They have no effect upon the results.
where $I_t$ is an index of the cost of living, less housing.

Figure 2 uses the non-housing component of the CPI for Singapore to chart the course of real investment returns in logarithms during the eleven-year period, from the three models. Although the mean returns differ by less than five basis points per month, the patterns of estimated returns and the estimated volatility from the three models are strikingly different.

Table 3 reports tests of the predictability of estimated monthly returns for the three indexes. We investigate the forecastability of returns based upon one-month and three-month lags. There is a consistent disparity in the predictability of investment returns implied by models based upon the three price generating processes. When spatial and serial correlations are recognized in the estimation of housing prices (Column A), there is no evidence that aggregate returns to housing price investment are predictable. However, when housing returns are estimated from a mean reverting process without spatial correlation (Column B), standard tests reject the null hypothesis of no predictability in returns. Finally, when returns are estimated from the conventional random walk model, they are strongly predictable. The p-values for both tests are less than 0.5 percent. These results are consistent for both a one-period and a three-period distributed lag.

The most striking feature of the table is that the predictability of the aggregate housing returns gradually disappear as restrictive assumptions on the individual housing price generating process are relaxed. When the assumption of the random walk with no spatial diffusion is

\[ R_t = \left( \frac{P_t + S_t}{P_{t-1}} \right) \left( \frac{I_{t+1}}{I_t} \right), \]

\footnote{We assume the implicit rent on owner-occupied condominiums, $S_t = 0.01P_t$ (See Englund, et. al., 2002).}

\footnote{These results are also apparent in more aggregated, quarterly price index models, not presented here. Indeed, the nonparametric kernel-based test (Hong, 1997) shows that the result does not rely upon any parametric specification of lag structure.}
maintained, p-values of the test statistics are very small. When the random walk assumption is relaxed (but spatial diffusion is not allowed), p-values are larger; the null hypothesis of no predictability in aggregate housing returns is still rejected at least for some of the tests. When returns on individual dwellings are allowed to be dependent over time and over space, p-values of the test statistics are all large enough that the null hypothesis of no predictability in aggregate housing returns is not rejected in any of the tests. This implies that the well known predictability in housing returns may arise simply because the underlying price index is inaccurately estimated due to restrictive assumptions about the price generation process.

Why does the aggregate return, when estimated with the random walk and no spatial diffusion assumption, appear to be significantly predictable while it exhibits no predictability when it is estimated without such restrictions?

To understand this, let \( \hat{R}_t = R_t^* + \zeta_t \), where \( \hat{R}_t \) is a regression-based estimate of the aggregate return, \( R_t^* \) is the true (unobserved) return, and \( \zeta_t \) is the estimation error. Consider a regression of the estimated aggregate return on its lagged term, \( \hat{R}_{t+1} = \beta_0 + \beta_1 \hat{R}_t + \nu_t \). The AR(1) coefficient, \( \beta_1 \), is computed as

\[
\beta_1 = \frac{\text{cov}(R_t^*, \hat{R}_{t+1}^*) + \text{cov}(\zeta_t, \zeta_{t+1})}{\text{var}(R_t^*) + \text{var}(\zeta_t)}
\]

For the autocorrelation coefficient, \( \beta_1 \), to be zero, it is required that: the true aggregate return be unpredictable, \( \text{cov}(R_t^*, \hat{R}_{t+1}^*) = 0 \); and the estimation error be not persistent, \( \text{cov}(\zeta_t, \zeta_{t+1}) = 0 \). Conversely, a non-zero estimate of the AR (1) coefficient, \( \beta_1 \), need not imply predictability in returns; it can arise from persistent forecast errors. This implies that any finding
that housing prices and housing returns are predictable may arise, simply by construction, if \( \text{cov}(\zeta_t, \zeta_{t+1}) \) is not equal to zero.

The properties of \( \zeta_t \) determine estimated autocorrelation coefficient, \( \beta_1 \). Suppose we have a sample of \( M \) houses transacted in each of \( T \) periods. If \( \text{cov}(R_t^*, R_{t+1}^*) = 0 \), it can be shown that

\[
(19) \quad E(\hat{\beta}_1) = \frac{E(\zeta_t, \zeta_{t+1}) + 0 \left( \frac{1}{T} \right)}{\sigma^2_e + \sigma^2 + 0 \left( \frac{1}{T} \right)} + \left\{ E(\zeta_t, \zeta_{t+1}) + 0 \left( \frac{1}{T} \right) \right\} 0 \left( \frac{1}{T} \right) - \frac{\sigma^2_e E(\zeta_t, \zeta_{t+1}) + 0 \left( \frac{1}{T} \right)}{\sigma^2_e + \sigma^2 + 0 \left( \frac{1}{T} \right)}^2,
\]

where \( E(\zeta_t, \zeta_{t+1}) \to 0 \) for large \( M \).

Equation (19) shows that \( E(\hat{\beta}_1) \) will converge to zero only if \( T \) is large enough and \( M \) is also large enough. The convergence of \( E(\zeta_t, \zeta_{t+1}) \) depends on the spatial structure of the housing market, and convergence is slow when prices of individual dwelling units are more highly correlated.\(^9\)

We can evaluate the impacts of the sample sizes of \( T \) and \( M \) by simulation. Using Singapore’s spatial structure, the previously estimated parameters, and assuming unpredictable aggregate returns, \( \beta_1 = 0 \), we simulate the Singapore private condominium market in two different dimensions, \( M \) and \( T \). First, we simulate the price of each house in the sample each month for ten years. We then randomly select some fixed number of houses each month, reflecting an underlying liquidity level or “sales frequency.” We use the sales so sampled to compute a repeat sales housing price index (Equation 2), incorrectly assuming a random walk

\(^9\) For example, when returns on individual dwelling units are not spatially correlated, its convergence speed is \( 1/M \). But when their prices are correlated, and their correlation is inversely related with distance, the convergence speed is \( \log(M)/M \).
and no spatial diffusion. These estimates are used to compute the index-based returns, the first order autocorrelation coefficient of housing returns, \( \hat{\beta}_1 \), and its t-ratio. We construct the base case of one percent sales probability, i.e., for each month one percent of total dwellings are randomly selected for trading. Then we extend the base case, first with a higher sales frequency of five percent, and second with a longer time series of fifty years and a one-percent sales frequency. Note that these two cases will generate the same number of observations.

Figure 3 summarizes these simulations, replicated 100 times. The figure reports the distributions of the estimated coefficient (\( \hat{\beta}_1 \)) and the corresponding t-statistics. It reports three distributions, the first with one-percent sales frequency and monthly observations for ten years, the second with the one-percent sales frequency but monthly observations for fifty years, and the third with five-percent sales frequency and monthly observations for ten years.

A sales frequency of one percent per month is much higher than the turnover rate observed in virtually all housing markets.\(^{10}\) A sales frequency of five percent per month is close to the turnover rate observed in the U.S. stock market.\(^{11}\) Thus, the one percent figure represents an illiquid market, and a five percent figure represents a very liquid market.

The probability distribution of \( \hat{\beta}_1 \) with one percent sales frequency and 120 monthly observations is sharply skewed to the left, centered around -0.3, quite far from the true value of 0. The distribution of t-statistics shows that usual t-tests are highly misleading. Among the 100 simulations, there are only fifteen instances where the t-statistic is larger than -2. This indicates that when the housing market has low sales frequencies, even though aggregate housing returns are not predictable, it is very likely that the estimated returns will be predictable. This apparent

\(^{10}\) The turnover rate in the Singapore market during the period was about 0.34 percent per month. The turnover rate for U.S. housing markets has averaged about 0.25 to 0.33 percent per month during the recent past. See Duca, 2005.
predictability persists for low values of the monthly sales frequency, and it is not eliminated until the monthly turnover rate reaches five percent level. When the sales probability is five percent, the distribution is substantially further to the right, and the center of the distribution is quite close to zero; the t-statistics are between -2 and 2 for 90 instances out of 100, and the null hypothesis of no predictability is rarely rejected using conventional tests.

However, when the sample period extends to fifty years (five times the original ten years) with the sales frequency kept at one percent, the results are quite different. The distribution moves to the right only by a small margin, and the center of the distribution is still well below -0.2. At the same time, dispersion of the distribution is reduced. In this instance, a small change in mean and a large reduction in variance shifts the distribution of t-statistics further to the left. Therefore, if the sales probability remains low, but the sample period is extended, the analyst is considerably more likely to reject the null hypothesis of no predictability. Indeed, there is no instance in the 100 simulations where the t-statistics is larger than -2.

This simulation indicates that it is important it is to have a large cross section of observations ($M$) to conduct a meaningful test of the predictability of aggregate returns. At the same time, the simulation also demonstrates that it is difficult to test for the predictability of returns in the housing market -- since the low frequency of housing sales is not due to inadequate data collection, but arises rather as an inherent feature of housing markets.

VI. Investment Performance

The results in Table 3 indicate that the aggregate housing return ($R_t = P_t - P_{t-1}$) is not predictable at the level of the aggregate housing index, but the idiosyncratic housing return

---

11 The weekly turnover rate of NYSE and AMEX during 1997-2001, reported in Cremers and Mei (2004), was 1.43%.
\((e_{it} - e_{it-1} = [V_t - V_{t-1}] - [P_t - P_{t-1}])\) is still predictable. Let \(\phi\) denote the serial correlation of monthly returns in an individual dwelling. It is straightforward to show, from (12), that

\[
\phi = -\frac{(1 - \lambda)\sigma^2_\mu + (1 + \lambda)\sigma^2_\eta}{2\sigma^2_\mu + 2(1 + \lambda)\sigma^2_\eta}.
\]

For the Singapore housing market, the results in Table 3 indicate that an individual housing return is substantially persistent, and its monthly serial correlation is -0.29. This has significant implications for investment in the local housing market. A better knowledge of the price process for individual dwellings can lead to superior investment decisions in two ways. First, improvement may arise through better estimates of aggregate housing price trends. Different assumptions about the price generating process have small effects on the large-sample properties of slope coefficients, but, as shown above, they do have substantial effects on the efficiency of the estimated aggregate returns when transaction frequencies are low. An investor who relies on random walk and no spatial diffusion would conclude that the aggregate housing return is predictable. Second, improved performance will arise from basing investment decisions on more complete information. For example, when housing prices are spatially and serially correlated, knowledge of past and present innovations in nearby dwellings provides information valuable for predicting the future course of prices for any house that the investor considers for investment.

The empirical issue is whether these signals are economically important.

To explore this, we conduct a second simulation of investor activity which utilizes the structure of a hypothetical “Grid City.” In this hypothetical city, dwellings are located on a square grid with 41 points on a side, and a house at each interior point is separated by 50 meters from its four nearest neighbors. Assume that the price of each dwelling follows the spatial and
temporal correlation process, as reported in Table 2 Column A (i.e., $\rho = 0.55$ and $\lambda = 0.89$). We simulate the price of each house in the Grid City each month for ten years. In the simulation, the true aggregate housing return is not predictable, $\beta_1 = 0$. We then analyze the results of investment rules which depend upon forecasts of future housing returns. The investment rule applied here is quite simple. Given assumptions on the price process and the consequent parameter values governing the processes, an investor makes forecasts for housing returns using all the available transactions information. The investor is instructed to “buy” if the forecasted return is greater than some preset threshold. When the investor decides not to buy, she is assumed to invest in some alternative asset that generates a risk free return. The threshold may be interpreted as some known transactions costs in the housing market. We set the risk free rate equal to zero for these simulations.

Transactions costs vary with housing market characteristics, financial market characteristics and tax systems, so it is difficult to specify a precise level.\textsuperscript{12} We use 0 percent, 6 percent and 12 percent as investment thresholds, comparable with a wide range of plausible transactions and opportunity costs.\textsuperscript{13}

The simulation of housing returns is performed in a similar manner to the previous section. The investment holding period is set at 24 months, 48 months, and 96 months. We assume that the investor observes the market and collects transactions information during an initial observation period. We set the observation period at 24 months, 48 months, 72 months and 96 months. For simplicity, we concentrate on the dwelling unit located at the center of the grid. A price for each of 1,681 dwelling units is generated for every month of the observation period.

\textsuperscript{12} The \textit{ex post} opportunity cost of housing investment in Singapore during the period 1990-2000 was in any case quite low. (Annual stock market returns averaged 0.1 percent; Treasury bill yields were about the same.)

\textsuperscript{13} For a more systematic examination of likely transactions costs in real estate, see Söderberg (1995) or Quigley (2002).
period and the holding period. From the simulated prices, a sample of houses is selected each month with the preset sales probability of one percent. Using the observations on the houses in the sample, together with her estimates of the parameters (\(\lambda\) and \(\rho\)), the investor makes a price forecast for the next 24, 48, or 96 months. If the forecasted return exceeds the threshold, she will invest. The price at the end of the holding period is then used to evaluate the return on her investment. We consider two investors with differing information. The fully informed investor is armed with the full knowledge of housing price dynamics which follows the correlation process reported in Table 2, column A, \(\rho=0.55\) and \(\lambda=0.89\). The uninformed investor forms her own forecasts in a similar manner. However, she assumes \(\rho=0\) and \(\lambda=1\), and does not recognize the serial and spatial correlation of prices. It is assumed that the transactions cost is paid when the house is sold, so the net performance of the investment is the capital gain less transactions cost.

Table 4 summarizes the forecast performance of the two investors. The table reports the average percent difference between the true price at the end of the holding period and the forecast made by the two investors. Each investor uses the information available in the observation period to make a forecast of the price at the end of the holding period. The forecast is compared to the actual price, and the average (absolute) percent deviations are reported in the table. The table reports the results of 2500 replications of this comparison using an underlying sales probability of one percent.

Clearly the percentage errors are larger when the forecast is for prices further in the future (that is, when the assumed holding period is longer). The percentage errors are likewise smaller when the forecast is based upon more information (that is, when the forecasts are based upon a longer period of observing property sales).
The results clearly establish that the informed investor makes better forecasts of future prices. For 15 out of 16 comparisons, the average error in the forecasts is less for the informed than for the uninformed investor. The difference is larger for shorter holding periods, but this advantage extends up to a holding period of eight years.

The economic significance of the small, but systematic, advantage of the informed investor in forecasting is analyzed in Table 5. Table 5 reports the increased returns, in percentage points, to the informed investor as a function of the observation period and the holding period. The average increased return to the informed investor is reported for 2500 replications with an underlying sales frequency of one percent. Results are reported for transactions costs of zero, six, and twelve percent.

With no transactions costs, the informed investor earns a return that is about 200 basis points higher than the uninformed investor. Even with high transactions costs, the fully informed investor outperforms the uninformed investor by one to four percentage points. Only when transactions costs are very high (twelve percent) and holding periods are very short (six months), does the uninformed investor perform almost as well as the informed investor.

VII. Conclusion

For the past fifteen years, it has been widely accepted that investment returns in housing are predictable. It is also widely believed that, due to high transactions costs, it is difficult to take advantage of this predictability. This paper develops a model of housing price determination that considers spatial correlation and serial correlation concurrently. Using comprehensive data on all Singapore condominium transactions, we estimate the extent of predictability in aggregate housing returns and in individual housing returns. The analysis
supports a general model of price discovery, rejecting a simple random walk model as well as a model with the mean reversion without spatial correlation. Importantly, when the appropriate error structure is taken into account, estimated aggregate housing returns are not predictable. Nevertheless, individual housing returns are still persistent -- that is, the housing return is only predictable at the level of the individual asset, not at the aggregate level. In contrast, when the aggregate housing price index is estimated from a random walk model, the estimated aggregate housing return shows substantial predictability. We show that this arises from the illiquid nature of housing transactions and the persistence of forecast errors in aggregate housing returns. The latter is due to spatial and temporal correlation in individual housing returns. Through extensive simulations, we also show that the pseudo predictability does not depend on some particular features of urban spatial structure.

Our simulation results suggest that an investor with enough information about the individual housing price process can, in fact, enjoy higher returns to housing investment. Our simulation results show that the investment performance of the fully informed investor is indeed superior to that of the naïve investor, even though her performance is bounded by holding periods and transactions costs.
References


Table 1. Summary of Sales Data on Singapore Condominiums
1990 – 2000

<table>
<thead>
<tr>
<th>Number of Times Sold</th>
<th>Number of Dwellings</th>
<th>Total Number of Sales</th>
<th>Price</th>
<th>Average Interval</th>
<th>Distance toCBD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Average*</td>
<td>Average Appreciation†</td>
<td>Std of Appreciation‡</td>
</tr>
<tr>
<td>1</td>
<td>42,169</td>
<td>42,169</td>
<td>861</td>
<td>31.98</td>
<td>129.93</td>
</tr>
<tr>
<td>2</td>
<td>8,791</td>
<td>17,582</td>
<td>913</td>
<td>0.52%</td>
<td>0.71%</td>
</tr>
<tr>
<td>3</td>
<td>1,195</td>
<td>3,585</td>
<td>1,030</td>
<td>0.68%</td>
<td>1.13%</td>
</tr>
<tr>
<td>4</td>
<td>190</td>
<td>760</td>
<td>1,087</td>
<td>0.73%</td>
<td>1.29%</td>
</tr>
<tr>
<td>5</td>
<td>28</td>
<td>140</td>
<td>1,418</td>
<td>0.92%</td>
<td>1.53%</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>24</td>
<td>1,129</td>
<td>0.86%</td>
<td>1.60%</td>
</tr>
</tbody>
</table>

* Thousands of current Singapore Dollars
** Number of months
† Average price appreciation between sales divided by average interval between sales in months.
‡ Standard deviation of price appreciation between sales divided by average interval between sales in months.
◆ Average number of months from sales to completion of construction of dwellings.
◊ Average size of dwellings in square meters.
№ Average distance in kilometers.
(t-statistics in parentheses)

<table>
<thead>
<tr>
<th>A. General Spatial-Temporal Process: $\lambda \neq 1$ and $\rho \neq 0$</th>
<th>B. Mean Reverting Process: $\lambda \neq 1$ and $\rho = 0$</th>
<th>C. Random walk Process: $\lambda = 1$ and $\rho = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.8877</td>
<td>0.7170</td>
</tr>
<tr>
<td>(287.84)</td>
<td>(62.022)</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.5512</td>
<td></td>
</tr>
<tr>
<td>(713.29)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>0.0535</td>
<td>2.82E-05</td>
</tr>
<tr>
<td>(14.340)</td>
<td>(0.0004)</td>
<td>(6.0016)</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>0.0691</td>
<td>0.1214</td>
</tr>
<tr>
<td>(46.606)</td>
<td>(53.760)</td>
<td>(2.4826)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0014</td>
<td>0.0009</td>
</tr>
<tr>
<td>(7.4748)</td>
<td>(6.2087)</td>
<td>(10.695)</td>
</tr>
<tr>
<td>Squared Interval</td>
<td></td>
<td>-1.26E-5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.1868)</td>
</tr>
<tr>
<td>Average of changes in price index</td>
<td>0.00384</td>
<td>0.00437</td>
</tr>
<tr>
<td>Std. Dev. of changes in price index</td>
<td>0.03530</td>
<td>0.04471</td>
</tr>
<tr>
<td>Loglikelihood</td>
<td>1112.31</td>
<td>277.00</td>
</tr>
<tr>
<td>Mean Square Error</td>
<td>0.05770</td>
<td>0.05708</td>
</tr>
</tbody>
</table>

Note: Estimates are based upon maximizing log likelihood function in Equation (16). The model also includes 132 time variables, one for each month between 1990 and 2000.

*: T-statistics are for the variances, not the standard deviations.
Table 3. Forecastability of Investment Returns, Singapore Condominiums, 1990-2000 (p-values in parentheses)

\[
\hat{R}_t = \beta_0 + \sum_{i}^{n} \beta_i \hat{R}_{t-i} + \nu_i
\]

<table>
<thead>
<tr>
<th></th>
<th>A. General Spatial-Temporal Process:</th>
<th>B. Mean Reverting Process:</th>
<th>C. Random walk Process:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\lambda \neq 1 \text{ and } \rho \neq 0)</td>
<td>(\lambda \neq 1 \text{ and } \rho = 0)</td>
<td>(\lambda = 1 \text{ and } \rho = 0)</td>
</tr>
<tr>
<td><strong>One Month Lag:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_1)</td>
<td>-0.0872</td>
<td>-0.1718</td>
<td>-0.2516</td>
</tr>
<tr>
<td>F-test for (\beta_1 = 0)</td>
<td>1.0395</td>
<td>4.4974</td>
<td>9.0974</td>
</tr>
<tr>
<td>(0.3099)</td>
<td>(0.0359)</td>
<td>(0.0031)</td>
<td></td>
</tr>
<tr>
<td>Box-Ljung Test</td>
<td>0.9706</td>
<td>3.9582</td>
<td>8.4758</td>
</tr>
<tr>
<td>(0.3245)</td>
<td>(0.0466)</td>
<td>(0.0036)</td>
<td></td>
</tr>
<tr>
<td><strong>Three Month Lag:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sum_{k=1}^{3} \beta_k)</td>
<td>0.2324</td>
<td>0.0326</td>
<td>-0.1765</td>
</tr>
<tr>
<td>F-test for (\beta_1 = \beta_2 = \beta_3 = 0)</td>
<td>2.3508</td>
<td>2.6531</td>
<td>3.5673</td>
</tr>
<tr>
<td>(0.0756)</td>
<td>(0.0516)</td>
<td>(0.0161)</td>
<td></td>
</tr>
<tr>
<td>Box-Ljung Test</td>
<td>4.6916</td>
<td>7.0204</td>
<td>12.0101</td>
</tr>
<tr>
<td>(0.1958)</td>
<td>(0.0713)</td>
<td>(0.0073)</td>
<td></td>
</tr>
<tr>
<td><strong>Nonparametric Test:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kernel-based test by Hong (1997)</td>
<td>0.8589</td>
<td>1.5906</td>
<td>4.2699</td>
</tr>
<tr>
<td>(0.3904)</td>
<td>(0.1117)</td>
<td>(0.0000)</td>
<td></td>
</tr>
</tbody>
</table>
Table 4. Difference in Forecasts by Fully Informed Investors and Uninformed Investors for Different Observation Periods and Holding Periods (2500 replications with sales probabilities of one percent per month)

<table>
<thead>
<tr>
<th>Observation Periods</th>
<th>Holding Period</th>
<th>6 month</th>
<th>24 months</th>
<th>48 months</th>
<th>96 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 months</td>
<td>Informed</td>
<td>15.85%</td>
<td>28.72%</td>
<td>46.68%</td>
<td>79.04%</td>
</tr>
<tr>
<td></td>
<td>Uninformed</td>
<td>15.98%</td>
<td>28.54%</td>
<td>47.16%</td>
<td>82.48%</td>
</tr>
<tr>
<td>48 months</td>
<td>Informed</td>
<td>14.34%</td>
<td>22.26%</td>
<td>33.24%</td>
<td>51.92%</td>
</tr>
<tr>
<td></td>
<td>Uninformed</td>
<td>15.68%</td>
<td>24.22%</td>
<td>34.16%</td>
<td>54.80%</td>
</tr>
<tr>
<td>72 months</td>
<td>Informed</td>
<td>13.91%</td>
<td>21.44%</td>
<td>28.92%</td>
<td>43.44%</td>
</tr>
<tr>
<td></td>
<td>Uninformed</td>
<td>15.42%</td>
<td>23.88%</td>
<td>30.64%</td>
<td>46.00%</td>
</tr>
<tr>
<td>96 months</td>
<td>Informed</td>
<td>13.83%</td>
<td>20.96%</td>
<td>27.76%</td>
<td>40.80%</td>
</tr>
<tr>
<td></td>
<td>Uninformed</td>
<td>14.96%</td>
<td>23.62%</td>
<td>30.12%</td>
<td>43.04%</td>
</tr>
</tbody>
</table>

Note: Entries in the table represent the average (absolute) percent difference between the true price at the end of the holding period and the forecast made by the informed and the uninformed investor.
Table 5. Economic Advantage of Fully Informed Investor over Uninformed Investors in Percentage Points.
(2500 replications with sales probabilities of one percent per month)

A. Transactions Cost: 0

<table>
<thead>
<tr>
<th>Observation Periods</th>
<th>Holding Period</th>
<th>6 month</th>
<th>24 months</th>
<th>48 months</th>
<th>96 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 months</td>
<td></td>
<td>2.21%</td>
<td>1.96%</td>
<td>1.52%</td>
<td>0.96%</td>
</tr>
<tr>
<td>48 months</td>
<td></td>
<td>2.89%</td>
<td>2.64%</td>
<td>2.16%</td>
<td>1.68%</td>
</tr>
<tr>
<td>72 months</td>
<td></td>
<td>2.80%</td>
<td>3.20%</td>
<td>2.24%</td>
<td>1.20%</td>
</tr>
<tr>
<td>96 months</td>
<td></td>
<td>2.42%</td>
<td>3.58%</td>
<td>2.08%</td>
<td>1.44%</td>
</tr>
</tbody>
</table>

B. Transactions Cost: 6 percent

<table>
<thead>
<tr>
<th>Observation Periods</th>
<th>Holding Period</th>
<th>6 month</th>
<th>24 months</th>
<th>48 months</th>
<th>96 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 months</td>
<td></td>
<td>1.77%</td>
<td>2.52%</td>
<td>2.12%</td>
<td>1.20%</td>
</tr>
<tr>
<td>48 months</td>
<td></td>
<td>1.68%</td>
<td>3.64%</td>
<td>2.28%</td>
<td>1.92%</td>
</tr>
<tr>
<td>72 months</td>
<td></td>
<td>1.92%</td>
<td>4.36%</td>
<td>2.64%</td>
<td>1.52%</td>
</tr>
<tr>
<td>96 months</td>
<td></td>
<td>1.57%</td>
<td>4.72%</td>
<td>3.32%</td>
<td>1.68%</td>
</tr>
</tbody>
</table>

C. Transactions Cost: 12 percent

<table>
<thead>
<tr>
<th>Observation Periods</th>
<th>Holding Period</th>
<th>6 month</th>
<th>24 months</th>
<th>48 months</th>
<th>96 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 months</td>
<td></td>
<td>-0.24%*</td>
<td>2.38%</td>
<td>2.36%</td>
<td>1.04%</td>
</tr>
<tr>
<td>48 months</td>
<td></td>
<td>0.13%*</td>
<td>3.48%</td>
<td>2.60%</td>
<td>2.00%</td>
</tr>
<tr>
<td>72 months</td>
<td></td>
<td>0.33%</td>
<td>4.30%</td>
<td>3.24%</td>
<td>1.84%</td>
</tr>
<tr>
<td>96 months</td>
<td></td>
<td>0.20%</td>
<td>3.84%</td>
<td>3.60%</td>
<td>1.92%</td>
</tr>
</tbody>
</table>

*: Statistically insignificant at 5%.
Figure 1.
Singapore Condominium Price Indices*
1990 – 2000

Note: *The figure graphs the monthly index values $I_t = \exp \left( \sum P_j \right)$ where the values of P are estimated from models reported in Table 2 by maximizing the likelihood function in equation (16).
Figure 2.
Monthly Returns to Investment in Singapore Condominiums
1990 – 2000
Figure 3. Empirical Distribution of AR(1) Coefficient and t-statistics
With different sales probabilities, in Singapore private condominium market