Abstract
The paper proposes a monetary model with nominal rigidities that differs from the conventional New Keynesian model in that firms set pricing policies instead of price levels. In response to permanent or highly persistent monetary policy shocks this model generates the empirically observed slow (inertial) and prolonged (persistent) reaction of the inflation rate, and also the recession which typically accompanies moderate disinflations. The reason is that firms respond to such shocks mostly through a change in the long-run or inflation updating component of their pricing policies. With staggered pricing policies this takes time to be reflected in aggregate inflation.

This paper was previously circulated under the title 'Macroeconomic Dynamics under Inflation Inertia: An Optimizing Model’. The authors thank Ariel Burstein, Guillermo Calvo, Chris Erceg, Andrew Levin and Zheng Liu for very helpful comments.
1 INTRODUCTION

A growing body of research in monetary theory uses the assumption of nominal rigidities embedded in dynamic general equilibrium models with rational expectations. Comprehensive surveys of this literature can be found in Galí (2001) and Lane (2001). The resurgence of this model class is based both on much improved theoretical foundations and on empirical arguments. The time-dependent price adjustment formulations of Taylor (1980), Rotemberg (1982) and Calvo (1983) made it possible to incorporate nominal rigidities into rational expectations models with forward-looking optimizing agents. Empirical support came from evidence showing that monetary policy has significant short-run real effects, such as Christiano, Eichenbaum and Evans (1996, 1998) and Leeper, Sims and Zha (1996).

Many authors\(^1\) argue that models with nominal rigidities can successfully account for most of the effects of monetary policy. But whether these models can fully account for all short-run empirical properties of inflation and output has recently been much debated. Mankiw (2001) notes that they do not generate the empirically observed delayed and gradual response of inflation to monetary policy shocks, a phenomenon that we will refer to as inflation inertia. Fuhrer and Moore (1995) show that they also do not generate the observed very prolonged steady state deviations of inflation following a monetary policy shock, a phenomenon that is generally referred to as inflation persistence. In short, these are models of stickiness in price levels, but they imply no stickiness in inflation. This in turn implies that disinflationary policies have minimal real costs, or even that anticipated disinflations cause

booms (Ball, 1994a). This is also inconsistent with a large body of empirical evidence (see e.g. Gordon 1982, 1997) which shows that disinflationary policies give rise to recessions, or more specifically to a U-shaped output response. These empirical regularities are typically presented using VAR impulse responses such as the ones displayed in Figure 1 for the US case, showing the response of the nominal interest rate, inflation and output to a one standard deviation monetary policy shock.²

![Figure 1: VAR Impulse Responses to a Monetary Policy Shock](image)

In this paper we use a tractable generalization of the Calvo (1983) staggered pricing model first introduced by Calvo, Celasun and Kumhof (2001, 2002). Our model contains the conventional staggered pricing model as a special case. But it is also capable of generating inflation inertia, inflation persistence and recessionary disinflations, and it does so without relying on nominal wage rigidities or real supply side rigidities. Its main difference to conventional treatments is in its specification of firms’ price setting behavior. We suggest

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² This is a recursive VAR with quarterly data from 1960:2 through 2000:4. The ordering and data are standard: Inflation (CPI growth rate), output (Hodrick-Prescott detrended real per capita GDP) and the interest rate (Fed Funds rate). The results are very similar to Stock and Watson (2001). The initial values shown for the interest rate and inflation are the sample averages.
that, in the realistic case of a positive steady state inflation rate, it is more plausible to assume that firms employ pricing policies instead of setting only a price level. The purpose of such policies is to keep them as close as possible to their steadily increasing flexible price optimum between the times at which price changing opportunities arrive. To keep the model tractable, we specifically assume that once a firm gets the chance to change its pricing policy, it jointly and optimally chooses an initial price level and an unconditional rate at which it will update its price in the future, a ‘firm-specific inflation rate’.

We motivate this specification by appealing to costs of reoptimization, such as costs of information gathering, decision making, negotiation and communication. The empirical evidence presented by Zbaracki, Ritson, Levy, Dutta and Bergen (2000) emphasizes the importance of reoptimization costs relative to menu costs (Akerlof and Yellen, 1985), the most common motivation for nominal rigidities. Christiano, Eichenbaum and Evans (2001) describe price setting behavior under reoptimization costs as follows: ‘...in the presence of these costs firms fully optimize prices only periodically, and follow simple rules for changing their prices at other times.’ In the existing literature there are two dominant approaches to specifying such a simple rule. In one (Woodford, 2002) firms choose only a price level without updating. In the other (Yun, 1996) firms still choose only a price level but update their prices at the steady state inflation rate at all times. But under both of these approaches only the aggregate price level is sticky while inflation is flexible. Credible disinflations therefore do not cause recessions.

By contrast, when firms employ pricing policies of the kind we propose, an unexpected and permanent decline in the steady state inflation rate targeted by monetary policy entails a slow inflation response and output losses, even if the change in policy is perfectly credible. There are two main reasons for this. The first is the continuing effect of historic pricing decisions. The economy initially contains a large number of firms that have chosen their price updating rates under the previous policy, and the weighted average of such updating rates is an important component of aggregate inflation. Intuitively, because it is costly for
firms to be continuously informed about monetary conditions, it takes time for their periodic inflationary updating to fully reflect the stance of monetary policy on inflation. The second reason for the slow inflation response is the behavior of new price setters. The spread between firms’ initially chosen price and the aggregate price level, or ‘front loading’, is the second component of aggregate inflation. Because firms have the option of updating their prices, front loading will respond very little to the policy change, contributing further to the sluggishness of the inflation response. Finally, the real interest rate increase induced by the slow inflation response gives rise to a recession.

The motivation for our pricing specification is similar to that of Mankiw and Reis (2002). These authors present a model where price setters are assumed to be able to reset their price every period, but receive information only at random intervals. This is equivalent to assuming that firms choose a price path, and it generates predictions that qualitatively match important features of the data. The drawback is that the model’s microeconomic foundations are not fully laid out, which makes it harder to explore its quantitative predictions and their sensitivity to the values of structural parameters.

The literature related to inflation inertia also encompasses models of backward-looking behavior, imperfect credibility, learning and supply side rigidities. Until quite recently the literature mostly relied on specifications that were not explicitly built on forward-looking optimizing behavior. Fuhrer and Moore (1995) present a relative real wage model, while Ghezzi (2001) and Clarida, Galí and Gertler (1999) modify the Calvo (1983) model to allow for a share of price setters to be backward looking, in the sense of using a rule of thumb that depends on lagged inflation. A well-known explanation for inflation inertia during disinflations is lack of credibility, see the papers by Ball (1995) and Calvo and Vegh (1993).

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3 See Calvo, Celasun and Kumhof (2001) for the original statement.
4 Burstein (2002) provides a general equilibrium model with microeconomic foundations that is related to Mankiw and Reis (2002). However, it is a nonlinear model and complex to solve. We will argue below that concentrating on linear pricing policies is both reasonable and advantageous for quantitative model evaluation.
However, in many countries where disinflations were costly the monetary authority enjoyed a high degree of credibility, as argued by Ball (1994b). This is therefore only a suitable explanation for a limited number of cases. Models of learning about monetary policy have recently become popular, and clearly such models do give rise to inflation inertia because the contain an element of backward-looking behavior. Two examples are Woodford (2001) and Erceg and Levin (2002). Christiano, Eichenbaum and Evans (2001) generate inflation and output inertia in a rational expectations model by introducing a number of nominal and real supply side rigidities. Their most successful model variant does however still rely on a backward-looking price and wage updating scheme.

The results of our paper will be presented as a comparison between our model and a conventional model with an identical demand side. We will refer to the latter as the Calvo-Yun model because we make the Yun (1996) assumption that firms update prices at the steady state inflation rate. As the only difference between these models is their price setting specification, the main differences in their performance arise under nominal shocks. The key point is that our model behaves very differently whenever aggregate inflation needs to change from its current level for a long period of time, because only then do firms have an incentive to change the long-run or inflation updating component of their pricing policies. The main example of such a shock is a highly persistent monetary policy shock, specifically a long-lasting change in the targeted inflation rate such as a disinflation. In that case our model displays inflation inertia and persistence and a U-shaped output response. On the other hand, under temporary monetary policy shocks the two models perform almost identically. This is because firms’ response to the shock will then be mostly through their current price level, just as in the Calvo-Yun model. Our approach is supported by the empirical evidence in Rudebusch (2002), who finds that monetary policy shocks are indeed highly persistent.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 compares the performance of the two models. Section 4 concludes.
2 THE MODEL

The economy consists of a continuum of measure one of identical price-taking infinitely-lived households, a continuum indexed by \( j \in [0, 1] \) of monopolistically competitive infinitely-lived firms, and a government.

2.1 Households

Households maximize lifetime utility, which depends on their per capita consumption \( \tilde{c}_t \), leisure \( 1 - L_t \) (where 1 is the fixed time endowment and \( L_t \) is total labor supply), and real money balances \( M_t/P_t \) (where \( M_t \) is nominal money and \( P_t \) is the aggregate price index). Households exhibit habit persistence with respect to \( \tilde{c}_t \), with habit parameter \( \nu.5 \) Consumption \( \tilde{c}_t \) is a CES aggregator over individual varieties \( \tilde{c}_t(j) \), with elasticity of substitution \( \sigma > 1 \). We scale consumption \( c \) and output \( y \) by the state of aggregate technology \( z \), and write \( c_t = \tilde{c}_t/z_t, \ c_t(j) = \tilde{c}_t(j)/z_t, \ y_t = \tilde{y}_t/z_t, \ y_t(j) = \tilde{y}_t(j)/z_t. \) Total scaled consumption \( c_t \) is therefore given by

\[
c_t = \left[ \int_0^1 c_t(j)^{\sigma-1} dj \right]^{\frac{\sigma}{\sigma-1}}.
\]  \hspace{1cm} (1)

The aggregate price index \( P_t \) is the consumption based price index associated with this consumption aggregator:

\[
P_t = \left[ \int_0^1 P_t(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}},
\]  \hspace{1cm} (2)

where \( P_t(j) \) is the price of variety \( j \). In addition to money households hold one period nominal government bonds \( B_t \) with nominal return \( i_t \). Their income consists of nominal wage income \( W_t L_t \), lump-sum profit redistributions from firms \( \int_0^1 \Pi_t(j) dj \), and lump-sum transfers from the government \( P_t \tau_t \).

\footnote{While habit persistence produces more reasonable output responses it is not essential for the main results.}
Households maximize the following objective function:

\[
Max \ E_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{u_{t+i}^{1-\frac{1}{\gamma}} - 1}{1 - \frac{1}{\gamma}} + \frac{a}{1 - \epsilon} \left( \frac{M_{t+i}}{P_{t+i}} \right)^{1-\epsilon} \right\}, \text{ where (3)}
\]

\[
u_t = (C_t)^{1-\kappa} (1 - L_t)^\kappa,
\]

\[
C_t = c_t z_t - \nu c_{t-1} z_{t-1},
\]

and where \( E_t \) is the expectation conditional on information available at time \( t \), and \( \gamma \) is the intertemporal elasticity of substitution. Households’ budget constraint is

\[
B_t = (1 + i_{t-1}) B_{t-1} + M_{t-1} - M_t + W_t L_t + \int_{0}^{1} \Pi_t(j) dj + P_t \tau_t - P_t c_t z_t. \text{ (4)}
\]

We denote the multiplier of this budget constraint by \( \Lambda_t \), and let \( \lambda_t = \Lambda_t P_t \). Then the first-order conditions with respect to \( c_t(j), c_t, L_t \) and \( B_t \) can be written as

\[
c_t(j) = c_t \left( \frac{P_t(j)}{P_t} \right)^{-\sigma}, \text{ (5)}
\]

\[
(1 - \kappa) u_t \frac{1-\frac{1}{\gamma}}{C_t} - \beta \nu (1 - \kappa) E_t u_{t+1}^{1-\frac{1}{\gamma}} = \lambda_t, \text{ (6)}
\]

\[
\kappa \frac{u_t^{1-\frac{1}{\gamma}}}{(1 - L_t)} = \lambda_t w_t, \text{ (7)}
\]

\[
\lambda_t = \beta (1 + i_t) E_t \left( \frac{\lambda_{t+1}}{1 + \pi_{t+1}} \right), \text{ (8)}
\]

where \( w_t = W_t/P_t \). Because the central bank will be assumed to follow an interest rate rule, the first-order condition for money is redundant. It simply determines the quantity of money required to meet the interest rate target, without affecting any other variables. We proceed to linearize conditions (6) - (8) around the steady state. A hat above a variable denotes its percent deviation from steady state, e.g. \( \hat{x}_t = (x_t - \bar{x}) / \bar{x} \), where \( \bar{x} \) is the steady state of \( x_t \). For interest and inflation rates it denotes the percent deviation of the gross rate from the steady state gross rate. Let \( \Gamma = (\bar{L}/(1 - \bar{L})) \).
Then the linearized first-order conditions are

\[
(1 - \frac{1}{\gamma}) \dot{u}_t - \dot{C}_t - \beta \nu \left(1 - \frac{1}{\gamma}\right) E_t \hat{u}_{t+1} + \beta \nu E_t \hat{C}_{t+1} = (1 - \beta \nu) \dot{\lambda}_t ,
\]

(9)

\[
\dot{\lambda}_t - \dot{\pi}_t = E_t \left(\hat{\lambda}_{t+1} - \hat{\pi}_{t+1}\right) ,
\]

(10)

\[
\hat{w}_t = \left(1 - \frac{1}{\gamma}\right) \hat{u}_t + \Gamma \hat{L}_t - \dot{\lambda}_t ,
\]

(11)

where

\[
\hat{u}_t = (1 - \kappa) \hat{C}_t - \kappa \Gamma \hat{L}_t ,
\]

(12)

\[
\hat{C}_t = \frac{1}{1 - \nu}(\hat{c}_t + \hat{z}_t) - \frac{\nu}{1 - \nu}(\hat{c}_{t-1} + \hat{z}_{t-1}) .
\]

(13)

### 2.2 Firms

Each firm \( j \in [0, 1] \) sells a distinct product variety. Heterogeneity in price setting decisions and therefore in demand for individual products arises because each firm receives its price changing opportunities at different, random points in time. Following Calvo (1983) it is assumed that these opportunities follow a geometric distribution, so that the probability \((1 - \delta)\) of a firm’s receiving a new opportunity is independent of how long ago it was last able to change its price. It is also independent across firms, so that it is straightforward to determine the aggregate distribution of prices.

We assume that firms’ unscaled output \( \tilde{y}_t(j) \) is produced via linear production functions in labor input \( l_t(j) \):

\[
\tilde{y}_t(j) = z_t l_t(j) , \text{ or } y_t(j) = l_t(j) .
\]

(14)

To simplify notation we assume a steady state productivity growth rate of zero and a steady state of \( \bar{z} = 1 \). We also assume the following law of motion for productivity:

\[
\hat{z}_t = \rho^z \hat{z}_{t-1} + \hat{\xi}^z_t .
\]

(15)
Firms have market power and therefore set the prices of their varieties \( P_t(j) \) to maximize profits taking into account consumers’ demand for their variety (5):

\[
y_t(j) = c_t \left( \frac{P_t(j)}{P_t} \right)^{-\sigma}.
\] (16)

In contrast to the Calvo-Yun pricing specification, we assume that when a firm \( j \) gets an opportunity to decide on its pricing policy, it chooses both its current price level \( V^j_t \) and the rate \( v_t^j \) at which it will update its price from today onwards until the time it is next allowed to change its policy. At any time \( t + k \) when the time \( t \) policy is still in force, its price is therefore\(^6\)

\[
P_{t+k}(j) = V^j_t (1 + v_t^j)^k.
\] (17)

Fuhrer and Moore (1995) discuss the role of aggregate inflation shocks. The counterpart to this notion in our structural model is an error in pricing across all firms \( \hat{\varepsilon}_\pi^t \), modeled as an equal percentage change in all firms’ updating factors \( (1 + v_{t-1}^j), j \in [0, 1] \).

Firms discount nominal profits expected in period \( t + k \) by the \( k \)-period ahead nominal interest rate \( i_t^k \) and by \( \delta^k \), the probability that their period \( t \) pricing policy will still be in force \( k \) periods from \( t \). The government is assumed to subsidize output at the rate \( \phi \) to eliminate the steady state markup distortion, following Rotemberg and Woodford (1998). Nominal revenue at \( t \) therefore equals \( P_t(j) y_t(j) z_t(1 + \phi) \). Labor markets are assumed to be competitive, with nominal wage bill \( W_t l_t(j) \). Firms’ problem is therefore

\[
Max_{V_t^j, v_t^j} E_t \sum_{k=0}^{\infty} \frac{\delta^k z_{t+k}}{1 + i_t^k} \left[ P_{t+k}(j) y_{t+k}(j)(1 + \phi) - \frac{W_{t+k}}{z_{t+k}} l_{t+k}(j) \right],
\]

subject to (14), (16) and (17). We substitute the constraints to get

\[
Max_{V_t^j, v_t^j} E_t \sum_{k=0}^{\infty} \frac{\delta^k z_{t+k}}{1 + i_t^k} P_{t+k} \left[ \left( \frac{V_t^j (1 + v_t^j)^k}{P_{t+k}} (1 + \phi) - \frac{w_{t+k}}{z_{t+k}} \right) \left( c_{t+k} \left( \frac{V_t^j (1 + v_t^j)^k}{P_{t+k}} \right)^{-\sigma} \right) \right].
\] (18)

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\(^6\) As for the possibility of more general price paths, we would argue that it is natural to focus on equilibria characterized by a constant steady state growth rate of the nominal anchor. The model can then be solved by linearizing around that steady state, in which case it is sufficient to allow firms to specify their pricing policies up to the growth rate of their price path. This permits the use of conventional solution methods, which makes quantitative analysis much more straightforward.
Note that the firm specific superscript \( j \) can be dropped because all firms that get a price changing opportunity at time \( t \) will behave identically. We now define the following terms:
\[ p_t \equiv \frac{V_t}{P_t} \] (the front loading term),
\[ \Pi_{t,k} \equiv \prod_{j=1}^{k} (1 + \pi_{t+j}) \] for \( k \geq 1 \) (\( \equiv 1 \) for \( k = 0 \)). The first-order condition with respect to \( V_t \) is then
\[ p_t = \frac{\sigma}{(\sigma - 1)(1 + \phi)} \frac{E_t \sum_{k=0}^{\infty} \frac{\delta^k z_{t+k}}{1 + \bar{\rho}^k} P_{t+k} \left( \frac{(1 + \nu_t)^k}{\Pi_{t,k}} \right)^{\sigma} c_{t+k} \frac{w_{t+k}}{z_{t+k}}}{E_t \sum_{k=0}^{\infty} \frac{\delta^k z_{t+k}}{1 + \bar{\rho}^k} P_{t+k} \left( \frac{(1 + \nu_t)^k}{\Pi_{t,k}} \right)^{1-\sigma} c_{t+k}} , \quad (19) \]
and with respect to \( v_t \) we have
\[ p_t = \frac{\sigma}{(\sigma - 1)(1 + \phi)} \frac{E_t \sum_{k=0}^{\infty} \frac{\delta^k z_{t+k}}{1 + \bar{\rho}^k} P_{t+k} \left( \frac{(1 + \nu_t)^k}{\Pi_{t,k}} \right)^{\sigma} c_{t+k} \frac{w_{t+k}}{z_{t+k}}}{E_t \sum_{k=0}^{\infty} \frac{\delta^k z_{t+k}}{1 + \bar{\rho}^k} P_{t+k} \left( \frac{(1 + \nu_t)^k}{\Pi_{t,k}} \right)^{1-\sigma} c_{t+k}} . \quad (20) \]

We set \( \phi = (\sigma - 1)^{-1} \) to eliminate the markup distortion, and therefore have \( \bar{w} = 1 \).

Before analyzing these conditions further we need to describe government policy and define equilibrium.

### 2.3 Government

The government’s fiscal policy is assumed to be Ricardian. In particular, we assume that the government budget is balanced period by period through lump-sum taxes/transfers, and that the initial stock of government bonds is zero. The budget constraint is therefore simply:
\[ \tau_t = \frac{M_t - M_{t-1}}{P_t} - \phi * y_t z_t \quad , \quad (21) \]
where we have used the fact that
\[ \int_0^1 P_t(j) y_t(j) dj = P_t y_t . \]
We assume that the central bank pursues the following interest rate rule for its policy instrument \( i_t \):
\[ (1 + i_t) = \beta^{-1}(1 + \bar{\pi}) \left( \frac{E_t(1 + \pi_{t+1})}{1 + \bar{\pi}} \right)^{\rho} \left( \frac{y_t}{\bar{y}} \right)^{\theta} (1 + h_t) \quad . \quad (22) \]
The first two components on the right-hand side equal the steady state gross nominal interest rate. The inflation target \( \bar{\pi} \) is an integral part of the specification of monetary

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\( ^7 \) The aggregate output index \( y_t \) is defined in the Appendix.
policy, and permanent monetary policy shocks will be modeled as permanent changes in \( \bar{\pi} \). The central bank interest rate response to expected deviations of inflation from its steady state value has the response coefficient \( \rho \). The response coefficient \( \theta \) applies to the ratio of technology-scaled aggregate output \( y_t \) to its steady state value \( \bar{y} \), or the output gap. In the present model this is equivalent to a central bank concern with unemployment. Finally, \( h_t \) is a zero mean monetary policy shock with law of motion

\[
\hat{h}_t = \rho^h \hat{h}_{t-1} + \varepsilon^h_t .
\] (23)

The rule (22) can be linearized as

\[
i_t = \rho E_t \hat{\pi}_{t+1} + \theta \hat{y}_t + \hat{h}_t .
\] (24)

Such forward-looking formulations of the policy rule are fairly common in theoretical work, but in empirical work it is more common to assume one of two other formulations. One is a backward-looking rule as in Rotemberg and Woodford (1998) or in Christiano, Eichenbaum and Evans (2001). The other is a forward-looking rule in inflation and output but adding a lag of the interest rate, also known as interest rate smoothing, as in Clarida, Galí and Gertler (1999). Our rule is closer to the latter but does not allow for interest rate smoothing. We replace this with the assumption that monetary policy can be characterized by persistent shocks \( \hat{h}_t \). This is motivated by the work of Rudebusch (2002), who shows that interest rate smoothing would imply a large amount of forecastable variation in interest rates at horizons of more than three months, which is contradicted by evidence from the term structure of interest rates. Highly persistent shocks are shown not to imply a large forecastable variation.

A government policy is defined as a set of stochastic processes \( \{i_s, \tau_s\}^{\infty}_{s=t} \) such that, given stochastic processes \( \{M_s, P_s, y_s, z_s, h_s\}^{\infty}_{s=t} \), the conditions (21) and (22) hold for all \( s \geq t \).
2.4 Equilibrium

A list of stochastic processes \( \{B_s, M_s, c_s, l_s, y_s, c_s(j), l_s(j), y_s(j), j \in [0, 1] \}_s=0^{\infty} \) is an allocation, with the relationships between \( c_s \) and \( c_s(j) \) and \( y_s(j) \) and \( l_s(j) \) given by (1) and (14), respectively. A price system is a list of stochastic processes \( \{P_s, W_s, P_s(j), V_s^j, v_s^j, j \in [0, 1] \}_s=0^{\infty} \), with the relationship between \( P_s \) and \( P_s(j) \) given by (2) and the relationship between \( P_s(j), V_s^j \) and \( v_s^j \) given by (17). Shock processes are a list of stochastic processes \( \{h_s, \varepsilon_s, z_s \}_s=0^{\infty} \). Then equilibrium is defined as follows:

An equilibrium given initial conditions \( h_{-1}, z_{-1} \) and \( P_{-1} \) is an allocation, a price system, a government policy and shock processes such that

(a) given the government policy, the price system and shock processes, the allocation solves the household’s problem of maximizing (3) subject to (4),

(b) given the government policy, shock processes, the restrictions on price setting, and the sequences \( \{P_s, W_s, c_s\}_s=0^{\infty} \), the sequences \( \{V_s^j, v_s^j, j \in [0, 1] \}_s=0^{\infty} \) solve firms’ problem of maximizing (18),

(c) the goods market clears for all goods and at all times, \( y_t(j) = c_t(j) \ \forall t, \forall j \in [0, 1] \),

(d) the labor market clears at all times, \( L_t = \int_0^1 l_t(j) dj \ \forall t \),

(e) the bond market clears at all times, \( B_t = 0 \ \forall t \).

The Appendix uses the definition of equilibrium to show that \( \bar{L} = \bar{c} = \bar{y} \), where \( \bar{L} \) is the proportion of time spent working in steady state, and that

\[
\bar{L}_t = \hat{c}_t = \hat{y}_t .
\] (25)

In equilibrium the nominal interest rate used by firms to discount future profits must equal the nominal interest rate entering households’ budget constraint and therefore their optimality condition (8). This implies that \((1 + \bar{i}^k)^{-1} = \beta^k E_t(\lambda_{t+k}/(\lambda_t \Pi_{t,k})) \). Therefore we have the following condition for firms’ steady state nominal discount factor: \( 1/(1 + \bar{i}^k) = (\beta/(1 + \bar{\pi}))^k \). We are now ready to linearize firms’ first-order conditions.
2.5 Linearized Price Dynamics

We linearize (19) for \( V_t \) and (20) for \( v_t \), quasi-difference them and combine them to generate a difference equation for \( \hat{v}_t \):\(^8\)

\[
(E_t \hat{v}_{t+1} - \hat{v}_t) = -\frac{(1 - \delta \beta)^2}{(\delta \beta)^2} (\hat{p}_t + \hat{z}_t - \hat{w}_t) \quad .
\] (26)

We combine this with the aggregate price dynamics derived from the index (2). Given our assumptions about price setting, that formula can be rewritten as

\[
P_t = \left[ (1 - \delta) \sum_{s=0}^{\infty} \delta^s [V_{t-s} (1 + v_{t-s})^s]^{1 - \sigma} \right] \frac{1}{1 - \sigma} .
\] (27)

Note that \( v_t \) is defined as the new firm-specific inflation rate from \( t \) to \( t + 1 \). This differs from the timing convention for the aggregate inflation rate from \( t \) to \( t + 1 \), which is \( \pi_{t+1} \). This convention is adopted because, unlike \( \pi_{t+1} \), \( v_t \) is known at \( t \) because the decision about it is taken at \( t \). We linearize and simplify (27), taking into account the aggregate inflation shock \( \hat{\varepsilon}_t^\pi \):

\[
\hat{\pi}_t = \frac{1 - \delta}{\delta} \hat{p}_t + (1 - \delta) \sum_{k=0}^{\infty} \delta^k \hat{v}_{t-1-k} + \hat{\varepsilon}_t^\pi .
\] (28)

We now define:

\[
\hat{\psi}_t = (1 - \delta) \sum_{k=0}^{\infty} \delta^k \hat{v}_{t-1-k} + \hat{\varepsilon}_t^\pi .
\] (29)

This is, in deviation form and accounting for inflation shocks, the weighted average of all those past firm-specific inflation rates that are still in force between periods \( t - 1 \) and \( t \), and which therefore enter into period \( t \) aggregate inflation. Note that \( v_t \) itself does not enter, because while it is determined at \( t \), it only starts to enter into aggregate inflation between \( t \) and \( t + 1 \). The variable \( \hat{\psi}_t \) is predetermined and follows the stochastic difference equation

\[
\hat{\psi}_t = \delta \hat{\psi}_{t-1} + (1 - \delta) \hat{v}_{t-1} + \hat{\varepsilon}_t^\pi .
\] (30)

Furthermore, we can use (29) to write (28) as

\[
\hat{\pi}_t = \frac{1 - \delta}{\delta} \hat{p}_t + \hat{\psi}_t .
\] (31)

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\(^8\) A separate appendix with complete derivations of this and other results is available from the authors upon request.
This is a key equation, because its two components reflect the two main sources of inflation inertia. Following a monetary policy shock, the continuing effects of price updating decisions made under the old monetary policy are represented by $\hat{\psi}_t$, and this is the main source of inertia in aggregate inflation. In addition, if a monetary policy shock is very persistent then new price setters respond mainly through changes in their updating rates. In that case front-loading $\hat{p}_t$ responds very little, thereby generating further inertia. Equation (31) allows us to rewrite (26) as

$$E_t \hat{v}_{t+1} = \left( \frac{\delta}{1-\delta} \left(\frac{1-\delta \beta}{\delta \beta}\right)^2 \right) \hat{\psi}_t + \hat{v}_t - \left( \frac{\delta}{1-\delta} \left(\frac{1-\delta \beta}{\delta \beta}\right)^2 \right) \hat{\pi}_t$$

and to obtain the following differential equation for $\hat{\pi}_t$:

$$E_t \hat{\pi}_{t+1} = \left( \delta (1+\delta) - \frac{2}{\beta} \right) \hat{\psi}_t + \left( (1+\delta)(1-\delta) \right) \hat{v}_t + \left( \frac{2}{\beta} - \delta \right) \hat{\pi}_t$$

To summarize, the dynamic behavior of the economy can be characterized by the aggregate demand block (9)-(13) and (25), the aggregate supply block (30), (32) and (33), the monetary policy rule (24), and the exogenous shock processes (15), (23) and $\hat{\varepsilon}_\pi^t$.  

### 2.6 The Calvo-Yun Model

The aggregate demand block, monetary policy and shock processes of the Calvo-Yun model are identical to the above. The aggregate supply block is replaced by the New Keynesian Phillips curve:

$$\beta E_t \hat{\pi}_{t+1} = \hat{\pi}_t - \Theta \hat{w}_t + \Theta \hat{z}_t - \hat{\varepsilon}_\pi^t$$

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\(^9\) We have established numerically that this system has a unique rational expectations solution for $\rho > 1$ but that it exhibits multiplicity for $\rho < 1$. For the current specification of preferences and technologies, the stability properties of our model are therefore the same as those of the conventional New Keynesian system. Namely, an aggressive Taylor rule ($\rho > 1$) gives rise to a unique solution while a passive rule ($\rho < 1$) gives rise to multiplicity.
where \( \Theta = (1 - \delta)(1 - \delta \beta) / \delta \). For this model we have, consistently with the treatment for our model, defined the inflation shock \( \hat{\varepsilon}_t^\pi \) as a serially uncorrelated shock to the updating rate of all firms that do not currently have a price changing opportunity.

3 MACROECONOMIC DYNAMICS

We calibrate parameter values for the quarterly frequency. The assumed value for the degree of price stickiness \( \delta = 0.75 \) implies an average contract length of four quarters, which is consistent with the empirical evidence (Taylor, 1998).\(^{10}\) The intertemporal elasticity of substitution \( \gamma \) is assumed to equal \( 0.5 \).\(^{11}\) The proportion of time spent working in steady state \( \bar{L} = 1/3 \) is based on the evidence cited in Kydland (1995). The value for the habit parameter \( \nu = 0.7 \) follows Boldrin, Christiano and Fisher (2001). We follow the literature in assuming \( \beta = 0.99 \). Our parameter choices for the monetary policy rule are \( \rho = 1.5 \) and \( \theta = 0.6 \). This is within the range of parameter estimates reported by Rudebusch (2002) for forward-looking rules without interest rate smoothing. As for the shock processes, we choose a high persistence \( \rho^z = 0.95 \) for the technology shock, which is a common assumption in the real business cycle literature, see Cooley and Prescott (1995). The parameter \( \rho^h \) determines the persistence of monetary policy shocks. Because this is a key factor in our model, we will explore the sensitivity of our results to various values of \( \rho^h \).

We solve the model by the algorithm of King and Watson (1997), and use impulse responses to display the dynamic response of the economy to four shocks. We start by examining two kinds of monetary policy shocks, a permanent change in the targeted steady state inflation rate \( \bar{\pi} \) and non-permanent shocks \( \hat{\varepsilon}_t^h \). We then analyze inflation shocks \( \hat{\varepsilon}_t^\pi \) and technology shocks \( \hat{\varepsilon}_t^z \). In all figures the solid lines represent the results of our model while

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\(^{10}\) Note however that the interpretation of the empirical evidence is different under the assumptions of our model. This is because we assume that many observed price changes are not associated with an updating of information about aggregate shocks. This would require a larger \( \delta \), which would give rise to additional inflation inertia and persistence. We nevertheless assume \( \delta = 0.75 \) for both models.

\(^{11}\) See e.g. Hansen and Singleton (1996).
the broken lines represent those of the Calvo-Yun model. The exception is the panel for $\nu_t$ and $\psi_t$, which is of course only relevant for our model. For ease of presentation output and labor are normalized to one.

### 3.1 Monetary Policy Shocks

We first consider a permanent change in $\bar{\pi}$ from 10\% to 3\% per annum. This is close in magnitude to the Volcker disinflation episode, a classical example of the kind of recessionary disinflation whose explanation is part of the motivation of this paper.

The impulse responses in Figure 2 reveal the shortcomings of the conventional staggered pricing model mentioned in the Introduction. Because firms are assumed to immediately start updating their prices at the new steady state inflation rate, inflation instantaneously drops to its new target level. It is therefore neither inertial nor persistent. And because the ex-ante real interest rate never changes, consumption, output and employment remain flat, i.e. disinflations are not recessionary.

In our model inflation exhibits both a very gradual initial response, inertia, and a very prolonged deviation from its (new) steady state, persistence. This is first because of the continuing effect of pricing decisions taken under the old, higher inflation monetary regime, and second because front-loading responds very little. The latter is due to the fact that monetary policy has changed permanently, so that price setters prefer to respond through the long-run or updating component of their pricing policies. The inflation deviation from steady state and the high response coefficient $\rho$ to such deviations in the monetary policy rule imply that nominal interest rates initially stay very high, and more importantly that there is a steep rise in the real interest rate. This causes consumption, output and therefore labor demand to drop, i.e. we observe the recession that is associated with disinflations in the data. This in turn lowers the real wage, which exerts downward pressure on prices so that inflation begins to fall. At the same time the recession induces lower nominal interest rates through the monetary policy rule. The combination of these two effects starts to lower real interest
rates, and once this process is complete the recession ends and inflation drops to its new target. An output sacrifice is therefore unavoidable in bringing down inflation.

The key ingredient required to obtain this result is that following the shock aggregate inflation is expected to be much lower than its initial value for a long period of time. In that case firms have an incentive to change the long-run component of their pricing policies, thereby delaying the instantaneous response of inflation. This reason for inertia is different from the one that is commonly stressed in the literature, which relies on a slow response of marginal cost to shocks. In our model inflation is inertial and persistent despite the fact that marginal cost (the real wage) is perfectly flexible.

The importance of a prolonged expected change in aggregate inflation suggests that the effect of a highly persistent monetary policy shock should be almost identical to that of a permanent shock. In our second monetary policy experiment we therefore assume that the steady state inflation rate remains at 10% per annum but that $\rho^h = 0.99$. Figure 3 shows that for our model the economy indeed responds almost exactly like in the previous case. But the Calvo-Yun model now performs very differently, exhibiting not only prolonged inflation but also output deviations from steady state. This discontinuity arises because the presence of a discounting factor in the New Keynesian Phillips curve (34) makes the model behave very differently depending on whether there are persistent inflation deviations from steady state or whether the steady state itself has changed. This unappealing feature follows directly from the rigidity of the updating assumption in that model, and is therefore not shared by our model. Furthermore, the Calvo-Yun model implies neither an inertial response of inflation nor a U-shaped output response.

Figure 4 studies the degrees of inflation inertia generated by the two models in more detail. It uses the ratio of the impact jump in inflation to the size of the monetary policy shock as a proxy for the inertia or slowness of the inflation response to monetary policy shocks, with a smaller ratio corresponding to more inertia. The figure shows how this ratio changes with increasing degrees of monetary policy shock persistence $\rho^h$. We see that for
\( \rho^h \) above 0.85 inflation is far more inertial, or far less ‘jumpy’, in our model. Inertia in fact starts to increase as \( \rho^h \to 1 \). For lower degrees of shock persistence inertia is very similar between the two models. To understand this, we now briefly turn to Figure 5, which displays impulse responses to a more transitory monetary policy shock, with \( \rho^h = 0.7 \). Because in this case the expected change in aggregate inflation is of a short duration, price setters react mostly through the short-term or front-loading component of their pricing policies. As a result pricing is almost identical for the two models. Therefore the real effects are also very similar. We observe a recession that is both more shallow and shorter than for a highly persistent shock of equal impact size.

### 3.2 Other Shocks

In Figure 6 we study an inflation shock \( \hat{\varepsilon}_t^\pi \). For ease of comparison with the disinflation case we assume a steady state inflation rate of 3% p.a. and subject the model to a shock that drives the inertial component of aggregate inflation \( \psi_t \) to 10% p.a. Because this drives initial aggregate inflation to around 10%, the impulse responses for this experiment are almost identical to those of a permanent disinflation from 10%, i.e. Figure 2. Note that our model exhibits these very persistent inflation deviations following a serially uncorrelated shock. In the Calvo-Yun model we only observe a one-off blip in inflation, which affects the ex-post but not the ex-ante real interest rate for one period. There are therefore no real effects.

Figure 7 shows the effects of a one percent increase in \( z_t \), the level of technology or productivity. With \( \rho^z = 0.95 \) productivity thereafter takes some time to return to its steady state level. The real effects are dominated by the productivity shock itself and they therefore differ little between the two models. The increase in productivity results in both an increase in unscaled output \( y_t z_t \) (and consumption \( c_t z_t \)) and a decrease in labor demand. But despite the latter the real wage rises by over 1%. This is because the monetary policy response to unemployment, or equivalently to a negative output gap, is a lower nominal and therefore real interest rate, which reduces the marginal utility of wealth by more than the marginal utility of
leisure, see (7). Aggregate inflation stays very close to 10% throughout, so inflation inertia plays almost no role. As a result the two models perform almost identically. This illustrates the fact that the main differences in the two models’ performance arise under nominal shocks, because they only differ in their price setting specifications.

4 CONCLUSION

This paper presents a monetary model with nominal rigidities and maximizing, rational, forward-looking households and firms. It differs from conventional models in this class in one key respect - firms set pricing policies instead of price levels. The paper is motivated by some important shortcomings of conventional models, namely their inability to generate inflation inertia, inflation persistence and recessionary disinflations.

The model does generate all of these effects in response to highly persistent monetary policy shocks such as permanent changes in the targeted inflation rate. The channel for these effects in the model is the long-run or inflation updating component of firms’ pricing policies. This is distinct from another frequently stressed reason for inflation inertia and persistence, a slow response of marginal cost to shocks. Because that channel is still important when shocks are less persistent, we will in future work study the role of staggered wage setting policies in addition to staggered pricing policies.
Figure 2: Permanent Disinflation

Figure 3: Highly Persistent Monetary Policy Shock $\rho^h = 0.99$
Figure 4: Inflation Inertia as a Function of $\rho^h$

Figure 5: Transitory Monetary Policy Shock $\rho^h = 0.7$
Figure 6: Inflation Shock

Figure 7: Technology Shock
Appendix  Consumption - Labor Supply Relationship

We begin by defining the scaled aggregate output index $y_t$ as

$$y_t = \left( \int_0^1 y_t(j)^{\frac{\sigma + 1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma + 1}}. \quad (A.1)$$

From the definition of equilibrium we know that $y_t(j) = c_t(j)$ and therefore $y_t = c_t$. Then (5) gives us

$$y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\sigma} y_t. \quad (A.2)$$

Next we define the scaled aggregate output quantity index $\tilde{y}_t$, using (14) and the definition of equilibrium, as

$$\tilde{y}_t = \int_0^1 y_t(j) dj = \int_0^1 l_t(j) dj = L_t, \quad (A.3)$$

and the alternative price index $\tilde{P}_t$ as

$$\tilde{P}_t = \left( \int_0^1 P_t(j)^{-\sigma} dj \right)^{-\frac{1}{\sigma}}. \quad (A.4)$$

Then we can derive the following relationship from (A.1) - (A.4):

$$L_t = y_t = \left( \frac{\tilde{P}_t}{P_t} \right)^{-\sigma} y_t = \left( \frac{\tilde{P}_t}{P_t} \right)^{-\sigma} c_t. \quad (A.5)$$

This implies that the steady state relationship between labor supply, consumption and output is

$$\bar{L} = \bar{c} = \bar{y}. \quad (A.6)$$

Furthermore, (A.5) can be linearized as

$$\hat{L}_t = \hat{c}_t = \hat{y}_t. \quad (A.7)$$
References


