THE EVOLUTION OF ENDOGENOUS BUSINESS CYCLES

Roger E. A. Farmer

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Roger E. A. Farmer*
University of California, Los Angeles
Hong Kong Institute for Monetary Research

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Abstract

This paper distinguishes between two kinds of Endogenous Business Cycle models, and discusses the evolution from first generation EBC1 models to second generation EBC2 models. I argue that EBC1 models, which display dynamic indeterminacy, are part of the evolution of modern macroeconomics that has classical roots dating back to the 1920s. EBC2 models, which display steady-state indeterminacy, are a more radical departure from the classical Real Business Cycle model; they represent a return to one of the most important ideas to emerge from Keynes' (1936) *General Theory*; that high involuntary unemployment can persist as part of the steady-state equilibrium of a market economy.

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Department of Economics, UCLA, 8283 Bunche Hall, Box 951477, Los Angeles CA 90095-1477.
Email: rfarmer@econ.ucla.edu

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1. Introduction

In a special issue of the *Journal of Economic Theory*, Benhabib and Farmer (1994) introduced a representative agent business cycle model in which equilibria are indeterminate. Writing in the same issue of the journal, Farmer and Guo (1994) developed a discrete time analog of the Benhabib-Farmer model and added self-fulfilling non-fundamental stochastic shocks to beliefs. Farmer and Guo (1994) constructed rational expectations equilibria that are randomizations across the indeterminate equilibria of the perfect foresight economy and showed that these equilibria mimic the features of real world business cycles.

Farmer and Guo’s model is characterized by a propagation mechanism in which the persistence of business cycles arises endogenously as opposed to the Real Business Cycle (RBC) model in which persistence is explained by an exogenous autocorrelated shock to total factor productivity (TFP). Their work signalled an important departure from the conventional RBC model by demonstrating that business cycles may not be the efficient responses of rational agents to shocks to technology; instead, they may be inefficient fluctuations in employment and GDP, caused by shocks to the self-fulfilling beliefs of households and firms. The 1994 JET volume spawned a literature on Endogenous Business Cycles (EBC) that refined the Farmer-Guo paper by reconciling the Benhabib-Farmer model with a broader range of micro and macro stylized facts.¹

In the Benhabib-Farmer model there is a unique steady state and a continuum of equilibrium paths that converge to it. I call the class of models that exploit dynamic indeterminacy to explain business cycles, first generation EBC models, or EBC1 models for short.² In a recent series of books and papers (Farmer, 2006, 2008a,b, 2010a,b,d,e, 2011, 2012a,b,c) I have introduced an endogenous business cycle model in which there is not just dynamic indeterminacy, but also steady-state indeterminacy. In this model, there is a continuum of steady-state unemployment rates. I will refer to EBC models that exploit steady-state indeterminacy to explain business cycles, as second generation EBC models or EBC2 models for short.

In (1999), Benhabib and Farmer published a survey of the state the EBC1 literature as it then stood. Since that date, many important papers have been published in the field. It is not my goal, in this paper, to provide a comprehensive survey of this literature; a survey of that kind would require far more space than I have available here. Instead, this paper is designed to explain to the reader, the connections between the EBC1 literature and a newer generation of EBC2 models in which there is

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¹ The term *endogenous business cycles* has been used in a number of different ways in the literature. Here, I use it in the sense of Schmitt-Grohé (2000) to mean macroeconomic models in which the propagation mechanism arises as a consequence of ‘sunspot’ shocks to an indeterminate DSGE model. This is distinct from an earlier usage that refers to non-linear cycle theories. That literature dates back to Goodwin (1951) and includes more recent papers by Benhabib and Nishimura (1979) and Grandmont (1985).

² Benhabib and Farmer (1999) survey EBC1 models and discuss the issues related to dynamic indeterminacy and the mechanisms that generate it. Farmer (1999) explains how indeterminacy can arise in general equilibrium models and provides an accessible introduction to the topic.
indeterminacy of the steady state. EBC1 models display dynamic indeterminacy, a feature that explains how self-fulfilling beliefs act as an independent impulse to the business cycle. EBC2 models display steady-state indeterminacy, a feature that explains how high unemployment, caused by self-fulfilling beliefs, can persist for five or ten years at a time as it did during the Great Depression of the 1930’s and, more recently, following the Great Recession of 2008.

2. A Short Primer on the History of Macroeconomic Theory

This section places first and second generation Endogenous Business Cycles within the history of thought. I will argue that first generation EBC1 models are part of the evolution of modern macroeconomics that has classical roots dating back to the 1920s. Second generation EBC2 models are a more radical departure from the classical paradigm; they represent a return to the economics of Keynes’ (1936) General Theory in which there may be a continuum of steady-state unemployment rates.\(^3\)

For the past forty years, macroeconomics has been based on a version of general equilibrium theory described in Chapter 7 of Debreu’s (1959) book, Theory of Value. There are two components that differentiate modern macroeconomics from the Keynesian theory that preceded it. The first is that labor markets are always in equilibrium; this was introduced by Lucas and Rapping (1970). The second is that expectations are rational in the sense of Muth (1961); this was introduced by Lucas (1972). The switch from Keynesian economics to the general equilibrium models that followed, has been called the rational expectations revolution.

Lucas’ (1972) paper was about the role of money in business cycles. With the seminal papers of Kydland and Prescott (1982) and Long and Plosser (1983), the focus shifted to rational-expectations models in which money plays a secondary role. These two papers introduced the idea that business cycles are the efficient fluctuations of a competitive economy in response to exogenous persistent technology shocks. There is a sense in which macroeconomics, at this point, took a 180 degree turn. The theory, dubbed Real Business Cycle theory after the title of Long and Plosser’s (1983) paper, represented a return to classical ideas that had been developed by business cycle theorists in the 1920s.

Pigou’s work is an example of the classical approach. In Industrial Fluctuations (1929), he provided a rich verbal theory of business cycles in which there are many possible causes of business fluctuations. These include productivity shocks, labor disputes, monetary shocks, taste shocks and self-fulfilling revisions to beliefs. In the language of Frisch (1965) these were all impulses to the economic system that would be transmitted to the endogenous variables, GDP, consumption, investment and employment, by a propagation mechanism operating through the market system. Pigou and his

\(^3\) For a discussion of the distinction between Keynesian economics and the economics of Keynes, see Leijonhufvud (1966). For a concise, accessible treatment of the history of macroeconomic thought see Farmer (2010d).
contemporaries envisaged business cycles to be fluctuations of a self-stabilizing system around a stationary full-employment steady state.

The Great Depression shattered that conception. Keynes (1936) argued instead that the economy can get stuck in a state of involuntary unemployment and that any unemployment rate can be an equilibrium. Keynes did not view the macroeconomy as a self-stabilizing system. He worried that businesses may lose confidence and fail to invest enough to maintain full employment.

Keynesian economics became the orthodoxy until, in the 1970’s, it was replaced by the rational expectations revolution. Because Keynes did not explain how his system was to be reconciled with the microeconomics of demand and supply, his theory was vulnerable to the attack that it lacked microfoundations. When the world economy experienced a bout of simultaneous high inflation and high unemployment, macroeconomists gave up on Keynes and reverted to the classical economics of Pigou.

3. Business Cycle Theory After Lucas

Although the Real Business Cycle model of Kydland and Prescott (1982) and Long and Plosser (1983) is grounded in classical ideas, it is mathematically more sophisticated. Because the math was hard, the initial RBC model was simple. In place of the rich panoply of shocks that drive business cycles in Pigou’s work, in the RBC model it is driven by a single random shock; innovations to total factor productivity.

The next twenty five years were characterized by a research agenda in which the business cycle shocks of the 1920’s were brought back, one by one, into the classical model. The models developed over this period are referred to as DSGE, or dynamic stochastic general equilibrium models.

Like the RBC model, DSGE models have a general equilibrium core. They differ from it by adding nominal frictions as in Christiano, Eichenbaum, and Evans (2005), additional shocks as in Hall (1997) and Beaudry and Portier (2006) or by making small departures from the core assumptions that provide a richer propagation mechanism as in Farmer and Guo (1994). By the onset of the Great Recession in 2007, Smets and Wouters (2007) had managed to replicate the verbal theory of Pigou using the language of dynamic stochastic general equilibrium theory. They showed that a DSGE model, loaded up with enough frictions and multiple shocks, does a credible job of replicating the dynamics of post-war U.S. business cycles.

4 The EBC2 literature, for example Farmer (2006, 2008b, 2010b, 2012b), provides a microfoundation for the Keynesian concept of persistent involuntary unemployment. In contrast, the textbook New-Keynesian model, see for example, Gali (2008) or Woodford (2003), does not even contain a variable to represent unemployment and cannot account for persistent deviations of GDP from trend. For a discussion of how New-Keynesians have dealt with this criticism, (Gertler and Trigari, 2009; Gertler, Sala, and Trigari, 2008; Gali, Smets, and Wouters, 2010), see Farmer (2012a), who argues that the New-Keynesian paradigm is a degenerative scientific research program in the sense of Lakatos (1978).
4. Indeterminacy, Sunspots and Self-fulfilling Prophecies

First generation EBC1 models are based on an idea developed at the University of Pennsylvania in the early 1980’s with the work of Cass and Shell (1983), Azariadis (1981), and Farmer and Woodford (1984); that indeterminacy can be combined with self-fulfilling beliefs to create a positive model of business cycles. Indeterminacy acts as the propagation mechanism and shocks to beliefs, caused by non-fundamental uncertainty, act as the impulse.

Using the term ‘sunspots’ to refer to non-fundamental uncertainty, Cass and Shell (1983) were the first to show that sunspots can have real effects on consumption, even in the presence of a complete set of financial markets. Using the term, ‘self-fulfilling prophecies’ to refer to non-fundamental uncertainty, Azariadis (1981) was the first to show that non-fundamental shocks could be added to a dynamic stochastic general equilibrium model to drive business cycles. The models of Cass and Shell and Azariadis were two-period lived overlapping generations models with a finite number of determinate equilibria.5

Indeterminacy as a positive explanation of business cycles was first introduced by Farmer and Woodford (1984), (published later as Farmer and Woodford (1997)) who combined self-fulfilling prophecies with indeterminacy to generate a model in which sunspot shocks generate endogenous autocorrelated responses of GDP and employment. Up to this point, models of indeterminacy and sunspots, or self-fulfilling prophecies, were recognized as theoretical possibilities but, because they were constructed in static models or in models where agents live for only two periods, they remained unconnected with quantitative models of business cycles.

That changed with the 1994 JET volume in which Benhabib and Farmer (1994) showed that indeterminacy occurs in models that are similar to the RBC model and Farmer and Guo (1994) provided a model where, for the first time, sunspot models could be taken seriously as quantitative models of the business cycle.

5. Endogenous Business Cycles

The EBC1 models of Benhabib and Farmer and Farmer and Guo, were one step in the DSGE research agenda. EBC1 models showed how self-fulfilling shocks to beliefs can be introduced as one of the shocks in a classical model.6 As with other DSGE models that enhanced the RBC framework,
first generation EBC1 models represent employment fluctuations as small deviations from a unique full employment steady-state equilibrium. Because the economy is never far from a Pareto Optimal steady state, the welfare costs of business cycles in these models are small.\(^7\)

In 2008, with the Lehman Brothers bankruptcy, we arrived at a moment not unlike the stock market crash of 1929. The subsequent protracted experience of low growth and high unemployment suggests that the classical vision of a self-stabilizing economy is deeply flawed. The EBC2 research agenda developed in Farmer (2006, 2008a,b, 2010a,b,d,e, 2011, 2012a,b,c) provides an alternative vision of the economy that explains episodes of high and persistent unemployment as an equilibrium phenomenon. According to Farmer’s explanation, self-fulfilling beliefs can trigger permanent movements in economic activity.\(^8\)

The defining feature of EBC2 models is the assumption that households are not on their labor supply curve. In this sense, EBC2 models are following Keynes’ General Theory.\(^9\) But Farmer (2006, 2008a,b, 2010a,b,d,e, 2011, 2012a,b,c) goes beyond the General Theory by providing an explicit microfoundation that explains why households are not on their labor supply curve. The labor supply equation is missing because there are incomplete factor markets. By this I mean that there are no prices for the two independent inputs to a technology that describes how searching workers are matched with vacant jobs; instead, workers find jobs through random search.


Several recent papers drop the labor supply equation from an otherwise classical model but are not explicit about how the exclusion of this equation can be reconciled with the microfoundations of the labor market. Papers in this literature include Schmitt-Grohé and Uribe (2011, 2012) who study the welfare consequences of downward wage rigidity in an open economy model, Heathcote and Perri (2012) who build a model with multiple steady state equilibria in which there may be an endogenous collapse in house prices and Kocherlakota (2012) who studies the impact of a fall in land prices on unemployment in a model with incomplete factor markets.\(^10\) Although these papers do not provide an

\(^7\) Lucas (1987) showed that, in an RBC model, the welfare costs of business cycles are less than one tenth of one percent of steady state consumption. In DSGE models with added frictions, the welfare costs of business cycle fluctuations are also small (Gali, Gertler, and Salido, 2007).

\(^8\) Farmer (2012c) argues that the stock market crash of 2008 was triggered by self-fulfilling beliefs and that it was the crash in asset values that caused the Great Recession.

\(^9\) Keynes drops what he calls ‘Postulate II of classical economics’. By Postulate II, he means that: “The utility of the wage when a given volume of labour is employed is equal to the marginal disutility of that amount of employment”, (Keynes, 1936, page 5).

\(^10\) Kocherlakota (2012) uses the term, incomplete labor markets, to refer to models where there is a missing labor supply curve. Kocherlakota’s usage mirrors the concept of incomplete factor markets originating in Farmer (2006, page 12) without providing a specific microfoundation for the absence of a labor supply equation.
explicit theory to replace the equation they remove, they are all consistent with the microfoundations for missing factor markets provided in Farmer (2006, 2010b).


EBC2 models, like their first generation cousins, rely on the idea that DSGE models may have multiple indeterminate equilibria to explain real world phenomena. Unlike EBC1 models, second generation models display steady-state indeterminacy. This is a significant departure from the earlier literature. Whereas EBC1 models add an additional shock, self-fulfilling beliefs, to a classical model; the EBC2 models developed in Farmer (2008a,b,2010a,b,d,e,2011,2012a,b,c) provide a microfoundation to the Keynesian idea that there may be many equilibrium unemployment rates. This work recasts the central ideas from The General Theory (1936) in the language of dynamic stochastic general equilibrium theory.11

6. First Generation, EBC1 Models

The Benhabib-Farmer model was successful because it was simple and closely related to the RBC model that by 1994, had become the industry standard. The canonical RBC model (King, Plosser, and Rebelo, 1988) consists of five equations and three boundary conditions to explain the time paths of five variables; GDP $Y_t$, consumption $C_t$, capital $K_t$, labor supply $L_t$, and total factor productivity $S_t$.12 These five equations are,

$$Y_t = S_t K_{t-1}^{\sigma} L_t^{1-\sigma},$$

(1)

$$K_t = K_{t-1} (1-\delta) + Y_t - C_t,$$

(2)

11 Indeterminacy of the steady state is not the only way of generating persistent unemployment. Alternative theories include Phelps’ (1994) structuralist model of the natural rate of unemployment and Blanchard and Summers (1986, 1987) who suggest that the insider-outsider model of Lindbeck and Snower (1986) who suggest that the insider-outsider model of Lindbeck and Snower (1986) is a promising way to introduce persistence to the unemployment rate. Frydman and Goldberg (2011) have argued that persistence is caused by non-stationarity of the fundamentals that invalidates the rational expectations assumption. They argue instead for a move to what they call ‘imperfect knowledge economics’. For a collection of related articles that question the foundations of modern macroeconomics, see Frydman and Phelps (2013).

12 I am using the convention that subscript $t$ means that $X_t$ is in the date $t$ information set. Hence it is $K_{t-1}$ that enters the production function at date $t$.
\[
\frac{1}{C_t} = E_t \left\{ \frac{1}{1 + \rho} \left( 1 - \delta + \frac{aY_{t+1}}{K_t} \right) \right\},
\]

(3)

\[
C_tL_t^\gamma = (1 - a) \frac{Y_t}{L_t},
\]

(4)

\[
S_t = S_{t-1} \exp(e_t),
\]

(5)

and the three boundary conditions are given by,

\[
K_0 = \overline{K}_0,
\]

(6)

\[
S_0 = \overline{S}_0,
\]

(7)

\[
\lim_{T \to \infty} E_{t,T} \left\{ \left( \frac{1}{1 + \rho} \right)^T \frac{K_T}{C_T} \right\} = 0.
\]

(8)

Equation (1) is a production function, (2) is the capital accumulation equation, (3) is the representative agent’s Euler equation, (4) is the first order condition for labor and (5) describes the evolution of TFP as a geometric first order autoregressive process, hit by an iid innovation. The innovation to TFP has a distribution function \( D(\cdot) \) with mean 0 and variance \( \sigma^2 \),

\[
e_t \sim D(0, \sigma^2).
\]

(9)

The boundary conditions are the initial conditions for capital, (6) and TFP, (7), and the transversality condition, (8). The model has five parameters; these are the rate of time preference \( \rho \), the capital elasticity \( a \), the labor supply parameter \( \gamma \), the autocorrelation parameter \( \lambda \) and the standard deviation of the innovation to TFP, \( \sigma \).

The EBC1 model studied by Farmer and Guo (1994) has an almost identical structure to the canonical RBC model but it differentiates between the private technology

\[
Y_t = S_t A_t K_{t-1}^a L_t^{1-a},
\]

(10)

and the social technology,
\[ Y_t = S, K_{t-1}^\alpha L_t^\beta, \]  

(11)

where the two are related by the identity

\[ A_t = K_{t-1}^{\alpha-\gamma} L_t^{\beta-\gamma}. \]  

(12)

Here, \( K_{t-1} \) and \( L_t \) refer to the economy-wide average use of capital and labor and \( A_t \) is a productive externality. In a symmetric equilibrium, \( K = K \) and \( L = L \) at all dates. An equilibrium of the model is a time path for the variables which satisfies the dynamic equations (1)-(5), the initial conditions (6) and (7) and the transversality condition, Equation (8).

In addition to the parameters of the RBC model, the EBC1 model has two new parameters, \( \alpha \) and \( \beta \). Benhabib and Farmer (1994) studied a continuous time version of the EBC1 model, and they showed that their model has multiple equilibria when \( \beta - \alpha > 1 \). Each of these equilibria is associated with the same initial capital stock and the same initial value of TFP, but first period labor supply, first period consumption and first period GDP differ. The sequences of capital, labor, consumption and GDP associated with each of these equilibria is consistent with all of the dynamic equations of the model and with the transversality condition. Which equilibrium prevails is determined by the self-fulfilling beliefs of the agents in the model.

7. Second Generation, EBC2 Models

EBC2 models drop the assumption that the demand and supply of labor are equal and they replace it with an explicit model of unemployment based on the search and matching framework of Alchian (1970), Mortensen (1970a,b), Pissarides (1976) and Diamond (1982a,b, 1984). In these models there are not enough prices to allocate search inputs correctly between the search time of unemployed workers and the search time of the recruiting departments of firms. As a consequence, a purely competitive search model, where firms and workers take prices and wages as given, does not have enough equations to determine all of the unknowns. Typically, theorists solve this problem by adding a new equation, the Nash-bargaining equation, and a new parameter, the bargaining weight, to determine the wage.\(^{14}\)

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\(^{13}\) Since the degree of returns-to-scale is equal to \( \alpha + \beta \), indeterminacy requires that there are increasing returns-to-scale. The assumption of input externalities is a device to introduce this property into a competitive model while preserving the marginal productivity theory of distribution. A second, equivalent way, of bringing in increasing returns is to assume monopolistic competition and increasing returns at the level of the individual firm (Benhabib and Farmer, 1994).

\(^{14}\) Much of the literature on search and matching evolved independently from macroeconomic models with two notable exceptions by Andolfatto (1996) and Merz (1995). Both of these papers follow the search and matching tradition of assuming that the wage is set by Nash-bargaining between the firm and the worker, after a worker has been matched with a vacant job. Howitt and McAfee (1987) pointed out that this assumption is arbitrary and, if firms and workers take prices and wages as given, search and matching models possess a continuum of equilibria.
Incomplete factor market models are closed in a different way. Farmer (2006, 2010b) drops the Nash bargaining equation and assumes instead that firms produce as much as is demanded. Demand is determined by forward looking households who form a sequence of self-fulfilling beliefs about the value of their wealth. The equation that determines beliefs is an alternative independent equation that replaces the assumption that firms and workers bargain over the wage.  

8. EBC2 Models with and without Investment

In Farmer (2006, 2010b, 2012b) I embedded a search market into an asset pricing model where capital is fixed and cannot be reproduced. I chose that framework because I wanted to model the connection between the value of the stock market and the value of unemployment, a connection that is strong and structurally stable in the post-war period (Farmer, 2012c). In a one good model with produced capital, the production possibilities frontier between labor and capital is linear. Hence the relative price of capital is constant and there is no obvious analog of the value of the stock market.

In the model with non-reproducible capital, the value of a capital asset varies with expectations of future dividends. Although this leads to a model where there is an obvious analog of stock market valuation, it cannot easily be compared with the RBC model because it does not allow for investment.

In his Ph.D. thesis, Plotnikov (2013) estimates an incomplete factor markets model with reproducible capital. Since stock market wealth does not enter his model, Plotnikov assumes instead that households form beliefs about their permanent income using adaptive expectations as in Friedman’s (1957) work on the consumption function. As in Farmer (2002), the adaptive expectations assumption anchors beliefs and selects an equilibrium.

I will use Plotnikov’s second generation EBC2 model in this discussion because it has a similar structure to the RBC model. Unlike that model, it replaces the assumption that households are on their labor supply curves with the alternative assumption that equilibrium is selected by self-fulfilling beliefs of households about their permanent income.

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15 Farmer (2012b) shows that there is an isomorphism between the assumption that the model is closed by self-fulfilling beliefs and the assumption that firms and workers bargain over the wage. For every equilibrium in the bargaining model, there is a sequence of self-fulfilling beliefs that implements the same equilibrium. Similarly, for every sequence of self-fulfilling beliefs in the incomplete factor market model there is a, possibly non-stationary, sequence of bargaining weights in the bargaining model that implements the same equilibrium. Although these two theories are observationally equivalent, I have argued, (Farmer, 2012b), that the assumption that the asset markets are driven by self-fulfilling beliefs provides a more plausible explanation of what caused the Great Depression and the Great Recession than a model in which bargaining weights are non-stationary and subject to occasional big revisions.

16 Whereas Farmer (2002) adds adaptive expectations to a first generation EBC1 model to select an equilibrium, Plotnikov (2013) adds adaptive expectations to a second generation EBC2 model where there are multiple indeterminate steady-state equilibria caused by incomplete factor markets.
9. The Equations of the Plotnikov EBC2 Model

Plotnikov (2013) EBC2 model has the following characteristics. Output is produced from labor and capital by a large number of competitive firms. Firms are owned by a representative household that allocates output between consumption and investment and next period’s capital stock is determined by a standard capital accumulation equation. These assumptions lead to a model that has five equations in common with the RBC model and with first generation EBC1 models. It is closed by adding an explicit theory of the determination of beliefs.

The RBC model does not contain prices. But when the solution to the model is decentralized with competitive markets, the household’s labor allocation decision, Equation (4), can be split into two parts as follows,

\[ \omega_t = C_t L_t^\gamma, \quad (13) \]
\[ \omega_t = (1 - a) \frac{Y_t}{L_t}, \quad (14) \]

where \( \omega_t \) is the real wage. Equation (13) reflects the assumption that the representative household equates the slope of an indifference curve between leisure and consumption to the real wage. Equation (14) is the first order condition for the choice of labor by a competitive firm.

If we add the real wage as a variable, the RBC model explains the six variables \( K_t, L_t, C_t, Y_t, S_t \) and \( \omega_t \) as functions of the innovation to TFP, with six equations; Equations (1)–(3), (5) and Equations (13) and (14). The EBC2 model has equations (15)–(19) in common with the EBC1 model.

\[ Y_t = A_t S_t K_{t-1}^{\alpha} L_t^{1-\alpha}, \quad (15) \]
\[ K_t = K_{t-1}(1 - \delta) + Y_t - C_t, \quad (16) \]
\[ \frac{1}{C_t} = E_t \left\{ \frac{1}{1 + \rho} \frac{1}{C_{t+1}} \left( 1 - \delta + a \frac{Y_{t+1}}{K_t} \right) \right\}, \quad (17) \]
\[ S_t = S_{t-1} \exp(e_t), \quad (18) \]
\[ \omega_t = (1 - a) \frac{Y_t}{L_t} \]  

(19)

The model also retains the boundary conditions, given by,

\[ K_0 = \bar{K}_0, \]  

(20)

\[ S_0 = \bar{S}_0, \]  

(21)

\[ \lim_{t \to \infty} E_t \left\{ \frac{1}{1 + \rho} \frac{K_T}{C_T} \right\} = 0. \]  

(22)

But this gives only five equations to determine the six unknowns, \( K_t, L_t, C_t, Y_t, S_t, \) and \( \omega_t. \) The Plotnikov EBC2 model is missing Equation (13). Instead of assuming that the labor market is competitive, employment is determined in a search equilibrium. Households do not vary labor supply in response to changes in wages and interest rates as in the RBC and EBC1 models; instead, each household sends a fixed fraction of its members to look for a job in every period and variation in employment arises as a consequence of endogenous changes in the efficiency with which workers are matched with jobs.

10. Unemployment in EBC2 Models with Incomplete Factor Markets

The EBC1 model differs from the RBC model by introducing an externality, \( A_t, \) defined in Equation (12). The EBC2 model introduces a similar externality, but in EBC2 models it is not just the labor input of other firms that affects an individual firm’s productivity; it is the way that those firms allocate their workers between two different activities. This externality appears in the model because factor markets are incomplete.

To model the frictional costs of recruiting, assume that a representative firm with \( L_t \) workers, can allocate them to the activity of recruiting or production. If we let \( V_t \) be the number of recruiters and \( X_t \) the number of production worker, \( V_t \) and \( X_t \) are related to \( L_t \) by the equation.

\[ L_t = X_t + V_t. \]  

(23)

Now assume that every recruiter can hire \( q_t \) workers,
\[ L_t = q_t V_t, \]  
(24)

where \( q_t \) is taken as given by the representative firm but is determined in aggregate by the degree of congestion in the labor market. Using the definition of \( X_t \), we can express the output of the representative firm as,

\[ Y_t = S_t K_t^a X_t^{1-a}. \]  
(25)

Substituting (23) and (24) into (25) leads to the expression,

\[ Y_t = A_t S_t K_t^a L_t^{1-a}, \]  
(26)

where

\[ A_t = \left(1 - \frac{1}{q_t}\right)^{1-a}. \]  
(27)

In words, the externality, \( A_t \), is a function of the number of workers, \( q_t \), than can be hired by a representative worker assigned to the task of recruiting. The term \( q_t \) is taken as given by each firm, but it is determined in aggregate by the number of other firms who are trying to attract workers. The connection with aggregate recruiting activity is found by specifying a matching technology that relates aggregate hires to the aggregate number of recruiters, \( V_t \).

Farmer (2012b) adds a Cobb-Douglas matching function to this model to determine the number of workers that are hired when firms, in aggregate, allocate \( V_t \) workers to recruiting and when a measure \( 1 \) of workers looks for a job. By making the simplifying assumption that all workers are fired and rehired every period,\(^\text{17}\) he shows that \( q_t = 1/L_t \), and hence the externality \( A_t \) is given by the expression,

\[ A_t = \left(1 - L_t\right)^{-a}. \]  
(28)

\(^{17}\) In most models of unemployment, see the survey by Rogerson, Shimer, and Wright (2005), the number of unemployed workers appears as a state variable. Farmer (2010b, 2012b) assumes instead that labor is fired and rehired every period. I maintain that assumption here since it allows me to write a second generation, EBC2 model that is close to first generation, EBC1 models and to the canonical RBC model. Farmer (2011) develops a model that relaxes this assumption and shows that nothing of substance hinges on the simplification.
As in the EBC1 model of Benhabib and Farmer (1994), the term $A_t$ represents a labor market externality. In the EBC2 model, this is represented by Equation (28), where $\bar{L}_t$ is average employment by all other firms.

11. Closing the Plotnikov EBC2 Model with Adaptive Expectations

The models developed in Farmer (2006, 2008a,b, 2010a,b,d,e, 2011, 2012a,b,c), are closed by assuming that households form self-fulfilling beliefs about the value of their wealth. In Plotnikov (2013), there is no analog of stock market wealth, but households must still form expectations of their human wealth. To capture this concept, Plotnikov adapts Friedman’s concept of permanent income. As in Friedman (1957), those expectations are formed adaptively. And as in Farmer (2002), because the model has an indeterminate set of equilibria, adaptive expectations are also rational.

If we evaluate Equations (15)-(18) at a steady state, we are able to pin down a value for $\bar{S}$ which equals 1, and values of the ratios $C/K$, $Y/K$ and $\bar{C}/\bar{Y}$, which are given by the expressions

\[
\frac{C}{K} = \frac{\rho + \delta(1-a)}{a},
\]

\[
\frac{\bar{Y}}{\bar{K}} = \frac{\rho + \delta}{a},
\]

and

\[
\frac{\bar{C}}{\bar{Y}} = \frac{\rho + \delta(1-a)}{\rho + \delta}.
\]

But the steady-state real wage, $\bar{\omega}$ and steady employment, $\bar{L}$ cannot be found from these equations. Instead, the model is closed by assuming, as in Friedman’s work on the consumption function, that consumption, $C_t$ is proportional to permanent income, $Y_t^p$,

\[
C_t = \phi Y_t^p.
\]

Here, permanent income is defined to be the value of income that would be earned by the representative household in the absence of shocks.
Because permanent income and current income are the same in a non-stochastic steady state, the coefficient $\phi$ is constrained by Equation (31) to be,

$$\phi \equiv \frac{\rho + \delta(1-a)}{\rho + \delta}.$$  (33)

Under the adaptive expectations hypothesis, permanent income depends on current income and on the view of permanent income that households formed one period in the past.$^{18}$ That assumption leads to Equation (34),

$$Y_t^p = \left(Y_t^p\right)^\phi Y_t^{1-\theta} \exp\left(e_t^b\right).$$  (34)

The parameter $\theta$ measures the speed with which revisions to current income are incorporated into permanent income and $e_t^b$ is a belief shock that represents the optimism or pessimism of households. This shock has distribution $D(\cdot)$ with mean 0 and variance $\sigma^2_b$,

$$e_t^b \sim D(0,\sigma^2_b).$$  (35)

Finally, since $Y_t^p$ is a state variable, the model must be closed with the initial condition

$$Y_0^p = Y^p.$$  (36)

The complete EBC2 model consists of the dynamic equations (15)--(19), (32) and (34), the initial conditions (20), (21) and (36) and the transversality condition, (22).

12. Using EBC2 Models to Explain the Data

EBC2 models with incomplete factor markets display hysteresis.$^{19}$ When these models are hit by shocks, there exist transformations of the unemployment rate and the real values of consumption and

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$^{18}$ Since $A_t$ enters technology as an externality, the marginal productivity theory of distribution holds in this model. It follows that income, and GDP are equal in this model, as in a standard RBC model.

$^{19}$ Blanchard and Summers (1986, 1987) have argued convincingly, that unemployment is highly persistent and that persistence should be modeled by a dynamical system that displays hysteresis. Hysteresis means that a small perturbation of the initial conditions leads to a similar perturbation of the eventual steady state. In a system that displays hysteresis, the equilibrium is path dependent.
GDP that follow cointegrated random walks.\textsuperscript{20} I have argued, (Farmer, 2010a, 2012c) that this is exactly the behavior we see in the data.

For any set of initial conditions, equations (15)–(19), (32) and (34) define a unique dynamic equilibrium. But setting the shocks to zero and solving for the steady state yields one less equation than unknown. This indeterminacy of the steady state arises because although Equations (32) and (34) define a unique path for any set of initial conditions, they do not add information to help pin down the steady state. The steady-state value of (34) defines $\bar{Y}^P$ to be equal to $\bar{Y}$, and the steady-state value of (32) replicates the information from (31). The complete set of equations defines a system that is path dependent. In the absence of shocks, the economy would converge to a steady-state value of employment that depends on the initial belief about permanent income, $\bar{Y}_0^P$.

When nominal consumption, nominal investment and nominal GDP are divided by the money wage, the productivity trend and the inflation trend should both be removed, leaving a set of stationary time series. When a nominal series is divided by the money wage, I will say that it is measured in wage units.

Figure 1 plots the log of GDP per member of the labor force, measured in wage units on the left axis, and the log of a logistic transformation of the unemployment rate, measured on the right axis. These series are highly persistent and well described by cointegrated random walks.\textsuperscript{21}

Figure 2 breaks the GDP series down into the logs of consumption and investment, per member of the labor force, measured in wage units. The investment series is volatile at business cycle frequencies and well described by a stationary autoregressive process of order 2. The consumption series is well described by a random walk. The Plotnikov EBC2 model explains these data with a permanent income theory of consumption in which beliefs about permanent income follow a random walk.

The highly persistent behavior of these data is ignored by most recent macroeconomic models because the trends in employment, consumption and GDP are removed by passing the data through the Hodrick-Prescott filter. In RBC and EBC1 models the economy fluctuates around a unique natural rate of unemployment. This property implies that consumption and GDP should be connected, not only at business cycle frequencies, but also at medium and low frequencies.

\textsuperscript{20} The qualifier ‘transformations’ is necessary because a random walk is unbounded above and below. It is the logarithms of GDP and consumption and the logarithm of a logistic transformation of the unemployment rate that follows a random walk in this model. This result is established in Farmer (2011) and it requires one of two modifications to the model described in this paper. Either, preferences must be CRA and technology must be CES, or we must drop the assumption that labor is hired and fired every period (Farmer, 2011).

\textsuperscript{21} Beyer and Farmer (2007) argue that the low frequency properties of unemployment, the interest rate and inflation are inconsistent with the natural rate hypothesis (NRH) and Farmer (2012a) shows that a monetary model that drops the NRH fits monetary data better than a small-scale New-Keynesian model. For a summary of why the NRH is inconsistent with the data see Farmer (2010c).
Figures 1 and 2 show that GDP and consumption data, when measured in wage units, are still highly persistent and that the remaining low frequency movements in GDP and consumption move closely with unemployment. This contradicts standard theory which implies that these variables should be stationary. It is however, consistent with EBC2 models which explain low frequency movements in the unemployment rate with a model that displays hysteresis.

13. Conclusion

This paper has discussed the use of general equilibrium models that display indeterminate equilibria as positive models of real world phenomena. This idea originated at the University of Pennsylvania during the early 1980s with the work of Azariadis (1981), Cass and Shell (1983) and Farmer and Woodford (1984) and it evolved into the EBC agenda that explains business cycles as endogenous responses to self-fulfilling shocks to beliefs.

I have identified two generations of EBC models. First generation EBC1 models display dynamic indeterminacy and, in these models, many equilibrium paths converge to the same steady state. Second generation EBC2 models display steady-state indeterminacy and, in these models, there are many steady-state equilibrium unemployment rates.

In the last hundred years, there have been two major changes in the way that macroeconomists think about their subject. Both of them were triggered by real world events that were hard to understand in terms of contemporaneous economic theory. In the 1930’s, the Great Depression led to a shift from classical to Keynesian economics. The economy was no longer seen as a self-stabilizing system and instead, under the Keynesian system, persistent involuntary unemployment was recognized as a possibility.

In the 1970’s, with the occurrence of high inflation and high unemployment, macroeconomists reverted to classical ideas of the 1920’s; but those ideas were formulated in the language of dynamic stochastic general equilibrium theory. For twenty five years, from 1982 through 2007, we have been refining DSGE models until, in 2007, macroeconomics arrived at a point where we were able to recreate the classical economics of the 1920s using the mathematics of functional analysis. First generation EBC1 models were an important part of this research agenda.

The Great Recession of 2008 is another game changing event. Although the Recession was declared to be over by the NBER in June of 2009, the U.S. has since experienced more than thirty six consecutive months of unemployment above 8%. As of July 2012, there are no signs that the economy will recover soon.

One of the most important ideas to come from Keynes’ General Theory was that high unemployment can persist as an equilibrium phenomenon. Second generation EBC2 models provide a microfoundation to this idea, and just as EBC1 models were part of the DSGE agenda that provided a
microfoundation for the economics of Pigou (1929), EBC2 models provide a microfoundation for the economics of Keynes (1936). The idea that involuntary unemployment can persist as an equilibrium phenomenon is one that will gain more credence, the longer the current recession persists.
References


Figure 1. GDP and Unemployment

Figure 2. Consumption and Investment