DYNAMICS OF MARKET ANOMALIES AND MEASUREMENT ERRORS OF RISK-FREE INTEREST RATES

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Abstract

Two-market anomalies since the 2008 global financial crisis – the widespread failure of covered interest parity (CIP) in foreign exchange swaps and negative 30-year US dollar interest rate swap-Treasury spreads have been challenging for conventional asset pricing models. Using a three-factor non-Gaussian-term structure model for the US Treasuries, an estimated short-rate premium tends to move in tandem with the CIP deviations and negative swap spread. The dynamics between the premium and two-market anomalies are found to be cointegrated, suggesting a long-run equilibrium between them. As the premium is found to be empirically related to demand for Treasuries, including the Fed’s quantitative easing program and demand for safe assets, it may reflect a convenience yield embedded in the yield curve such that the observed Treasury interest rate is lower than the true risk-free interest rate. The anomalies manifest such measurement error as additional spreads on the observed US dollar interest rates for pricing the corresponding instruments, consistent with recent studies that the US dollar and its interest rates play an important role in determining the CIP deviations.

Keywords: covered interest rate parity, swap spreads, Treasuries, term-structure models; safe assets

JEL classification: F31, G13

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1. Introduction

Two market anomalies in the foreign exchange (FX) and interest rate markets since the 2008 global financial crisis (GFC) have attracted the attention of market participants, policy makers and researchers. The anomalies are the widespread failure of covered interest parity (CIP); and US dollar (USD) interest rate swaps with a thirty year maturity below Treasury rates with the same maturity, for example, a negative swap spread. Both phenomena are challenging for typical asset pricing models as they seem to imply a risk-free arbitrage opportunity under standard assumptions. Figure 1 shows the CIP deviations exhibited by the 1-year cross-currency swap basis (quoted as a spread over USD LIBOR) of the Japanese yen (JPY), euro (EUR) and Swiss franc (CHF) versus the USD; and the spread between the 30-year interest rate swap and Treasury, indicating similar patterns of movements. The cross-currency basis is the difference between the USD interest rate in the cash market (LIBOR) and the USD interest rate implied from the swap market when swapping foreign currency into USD.

A CIP deviation is illustrated by the interest parity theory, which states that the equilibrium forward exchange rate \( F \) is:

\[
F = \frac{S(1 + q)}{(1 + r)},
\]

where \( S \) is the spot exchange rate (the foreign currency value of a unit of USD), and \( q \) and \( r \) are the foreign and US rates of interest on securities respectively, which are identical in all respects except for the currency of denomination. Eq.(1) states that the return of investing a sum of money in a domestic interest-bearing asset for a certain period of time is the same as the return of investing in a similar foreign interest-bearing asset by converting the sum into a foreign currency while simultaneously buying a forwards contract to exchange the investment back to the domestic currency at the end of the period, identified as a CIP condition. If the returns are different, an arbitrage transaction could produce a risk-free return, for example, a deviation from CIP. In other words, if the party lending a currency via an FX swap makes a higher or lower return than implied by the interest rate differential in the two currencies, then CIP fails to hold. By rearranging Eq.(1) and introducing the FX swap-implied
USD rate $r^*$, this means:

$$\frac{S}{F} (1 + q) - (1 + r) = r^* - r,$$

(2)

such that $(r^* - r)$ is the FX-swap spread that represents the premium or discount as reflected in the swap-implied USD funding rate. If CIP holds, $(r^* - r) \approx 0$.

It is important to note that CIP assumes assets denominated in domestic and foreign currencies are freely traded internationally (i.e., no capital controls) and have negligible transaction costs and similar risks. Given today’s market structures and technology, these assumptions normally hold in the international financial markets. Therefore the parity condition is observed almost all the time (except for those countries where capital controls are still in place). In view of these arguments, CIP is considered the closest thing to a physical law in international finance (see Financial Times https://ftalphaville.ft.com/2016/06/09/2165690/textbook-defying-global-dollar-shortages). However, since the onset of the GFC, Figure 1 shows the persistence of cross-currency swap bases for the JPY, EUR and CHF versus the USD since 2008. A non-zero cross-currency basis indicates a deviation from CIP. Initially, both counterparty risk and funding liquidity risk which inhibited arbitrage in the European banking systems were the significant determinants of such deviation identified by Baba and Packer (2009) and Hui et al. (2011) during the period of 2008-2009. But, the deviations have persisted and even increased after these constraints of arbitrage in the banking system dissipated. Du et al. (2016), Liao (2016), Iida et al. (2016), Sushko et al. (2016), Wong et al. (2016) and Avdjiev (2016) attempt to explain the reasons behind the persistent CIP deviations. They propose the strength of the USD and associated hedging demand, supply of dollar funding and associated counterparty risk, banks’ balance sheet structure, and asymmetric monetary policy shocks in particular normalisation in the US, are the main drivers of CIP deviations. Most of these drivers have a common factor, that the USD and its interest rates play an important role in determining the CIP deviations.

The second market anomaly is in USD interest rate swaps. Since October 2008 the swap spreads between fixed rates for interest rate swaps and Treasury rates with the same maturity have fallen

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1 Taylor (1989) finds that, during the float of the sterling in 1972 and the inception of the European Monetary System in 1979, significant departure had occurred from CIP for periods long enough to challenge the theory. Other studies have attempted to rationalise these departures in terms of transaction costs, for example, Frenkel and Levich (1977) and Clinton (1988).
substantially, in particular the 30-year swap spread, which has been mostly negative, as shown in Figure 1. The negative swap spread represents a risk-free arbitrage with a positive cash flow by investing in a Treasury bond and paying the lower fixed swap rate until maturity, assuming the funding cost for the bond is the same as receiving USD LIBOR in the swap. While capital requirements and funding liquidity make such an investment risky and thus limit arbitrage, sophisticated investors can use repo agreements to finance Treasuries and roll them over for long periods of time, which reduces the capital and funding constraints. Moreover, these constraints of arbitrage in the market have relaxed since 2013 due to a more stable global financial system, but the negative swap rate has persisted. Klinglery and Sundaresan (2016) show the demand for swaps arising from duration hedging needs of underfunded pension plans can cause negative swap spreads. Jermann (2016) develops a model to show frictions for holding long-term bonds as an explanation.

Different from the recent studies on these two market anomalies, this paper shows that a latent factor embedded in the US Treasury yield curve displays a similar movement with the CIP deviations and 30-year swap spread over time since 2009. The dynamics between this latent factor and the two market anomalies are found to be cointegrated, suggesting a long-run equilibrium between them. The latent factor is estimated from a three-factor non-Gaussian term structure model of bond yields. The model has closed-form solutions for bond prices that are functions of a small number of unobservable state variables. The first state variable in the model is the instantaneous short-term interest rate (short rate). It follows the Cox-Ingersoll-Ross (CIR) (1985) model, which is a general equilibrium model and preserves the non-negativity of interest rates. The second variable is a stochastic long-term mean to which the short rate reverts. This approach follows Balduzzi et al. (1998) assuming that the short rate and long-term mean are coupled stochastic processes. Treasury yield curves in the model depend on these two state variables which have natural interpretations in terms of fundamentals. In principle, the coupled dynamics of the short rate and long-term mean can reflect observations on macro variables such as expected inflations. Therefore, the short rate estimated from the model should represent the instantaneous short-term risk-free rate consistent with fundamentals anticipated in the entire yield curve. The third state variable is a latent factor that is found to be cointegrated with the dynamics of the two market anomalies. It is a short-rate-premium factor that captures macro information not already contained in the other two state variables. This state variable enters into the model such that the observable short-term Treasury yield is simply the sum of it and the short rate.
The short-rate-premium factor in the term structure model likely captures the information related to demand of Treasuries due to their safety and liquidity which is not captured in the short-rate process. Recent studies show Treasuries carrying a convenience yield of holding them, which could be reflected in the short-term premium. Investors are willing to forgo some interest (a convenience yield) in exchange for owning a high-liquid and safe debt instrument, in particular Treasuries. An important implication of the convenience yield, as stated by Krishnamurthy and Vissing-Jorgensen (2012), is that the common practice of identifying the Treasury yield with asset pricing models’ risk-free interest rate is incorrect. The observed Treasury interest rate is lower than the “true” risk-free interest rate. A number of studies have quantified convenience yields in the data and have documented their significant component of equilibrium bond prices (for example, Krishnamurthy (2002), Longstaff (2004), Fontaine and Garcia (2012), Krishnamurthy and Vissing-Jorgensen (2012), Smith (2012), Greenwood and Vayanos (2014), and Valchev (2016)). The average (annualised) convenience yield on Treasuries ranges between 75 and 166 basis points (bps), and the estimates of the standard deviation range between 45 and 115 bps. Related theoretical literature has explored bond convenience yields as a possible explanation for asset pricing puzzles, such as the equity risk premium and the term premium (for example, Bansal and Coleman (1996), Lagos (2010), Bansal et al. (2011), Acharya and Viswanathan (2011)). Del Negro et al. (2017) identify a rise in the measured convenience yield as a key driver of the secular decline in the natural rate of interest. The demand for Treasuries was enhanced by the US Fed’s ultra-accommodative monetary policy by introducing quantitative easing policy including purchasing Treasury bonds and operation twist after the GFC, which lowered long-term borrowing costs and fostered economic activity. The two market anomalies have coexisted with the ultra-accommodative monetary policy. We provide empirical evidence that the short-rate premium captures demand for the Treasuries after the GFC. In view of this empirical evidence as well as the dynamic relationship between the short-rate premium and the two market anomalies, we suggest that the anomalies manifest a measurement error of risk-free interest rates. Both the FX swap and interest rate swap markets correct this measurement error by adding spreads on USD LIBOR rates that follow short-term Treasury interest rates. As reflected in the market prices,

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2 Treasuries also serve as common collateral in financial transactions. There is a special demand of Treasuries for safe assets, from mutual funds or insurance companies mandated to invest in certain class of assets.

3 The Fed lowered the policy interest rate to 0–0.25% in 2008 (see Figure 2). “Operation Twist” describes a monetary process where the Fed sells short-term and buys long-term bonds.
spreads are added on the USD LIBOR rates in cross-currency basis swaps and lower fixed rates are priced in interest rate swaps. In other words, the FX swap and interest rate swap markets have priced in a correct USD risk-free interest rate in actual transactional prices.

The next section provides the derivation of the three-factor, non-Gaussian term structure model of bond yields. In section 3, the results of the model are presented. The relevance of the model results to the CIP deviations and negative 30-year swap spread, and the empirical analysis on the short-rate premium and demand for Treasuries, are discussed in section 4. The final section concludes.

2. Yield curve model with short rate and short-rate premium

We assume a state vector $X_t = (r_t, \theta, L)$, where $r$ is the risk-free short interest rate, $\theta$ is the mean of $r$, and $L$ captures the premium on $r$ (i.e., value of safety and liquidity of Treasuries). All variables are unobservable but can be inferred from yields through the bond-pricing model. We propose a three-factor, non-Gaussian affine term structure model. The three factors are assumed to be uncorrelated to each other. Specifically, the instantaneous short rate $r$ is described by the following mean-reverting square-root process:

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}dZ_t,$$  \hspace{1cm} (3)

where $\kappa$ determines the speed of the mean-reverting drift towards the long-term mean $\theta$, $\sigma$ is the volatility and $dZ_t$ is a standard Brownian motion. The short-rate dynamics in Eq.(3) follow the general equilibrium CIR model. An advantage of the CIR model is that the risk-free short rate and its dynamics are determined endogenously as part of the general equilibrium. The model can deal with a real economy by introducing some aspects of money and inflation such that one of the state variables represents a price level and that some contracts have payoffs whose real value depends on this price level. The general theory in Cox et al. (1985) shows that the nominal short-term interest rate, which can be expressed in terms of the real interest rate and the expected inflation rate, may be either

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4 Chung et al. (2014) use a similar model to evaluate the effectiveness of the date-based forward guidance at the zero lower bound.
greater or less than the sum of them. By using the well-known Taylor rule which is a simple equation intended to describe the interest rate decisions of the Federal Reserve’s Federal Open Market Committee, the interest rate implied by the rule related to the current state of the economy shown in Figure 2 is higher than the fed funds target rate and 1-month Treasury yield since 2011. The Taylor-rule implied rate suggests that the estimated short rate by the model is probably higher than the 1-month US Treasury-bill yield, which fell to the zero lower bound for extended periods of time until the Fed starting rate hike in December 2015. This is consistent with the dynamical specification of the short rate in the CIR model, which is guaranteed to remain non-negative, if \( 2\kappa \theta > \sigma^2 \) in Eq.(3).

The long-term mean \( \theta \) of the short rate in turn follows another mean-reverting square-root process:

\[
d\theta_t = \alpha(\beta - \theta_t)dt + \eta \sqrt{\theta_t}dZ_{\theta_t}
\]

where \( \alpha, \beta \) and \( \eta \) are the mean-reversion parameters, long-term mean, and volatility of the Brownian motion respectively. Eqs.(3) and (4) together are sometimes referred to as the stochastic mean model. This specification follows Balduzzi et al. (1998) to model the instantaneous short rate and its long-term mean level as a coupled stochastic process. The stochastic mean model can resolve the undesirable tendency of reverting back to a higher mean level as predicted by a standard one-factor CIR model. Specifically, it is well known that the variance of a square-root process will become smaller when the short rate is close to zero, and that the evolution of the short rate will be largely dictated by the drift term. As a result, in the standard CIR model which assumes a constant mean level for the short rate, the constant drift term tends to pull the short rate back to its higher long-term mean level when the short rate is near zero. In the stochastic mean model, the short rate can remain near the zero bound if the long-term mean level \( \theta \) is also low.

Finally, there is a factor \( L \), which follows the stochastic process as:

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5 The Taylor rule’s formula (Taylor, 1993) is:

\[
R = p + 0.5y + 0.5(p - 2) + 2\%
\]

Where

- \( R \) = the federal funds rate
- \( p \) = the rate of inflation
- \( y \) = the percent deviation of real GDP from a target

6 See Karlin and Taylor (1981. Ch. 15).
where $\xi$ and $\gamma$ are the drift term and volatility scalar parameter of the Brownian motion $dZ_L$ respectively. The process is a special case of the constant elasticity of variance (CEV) model with the constant diffusion coefficient and the instantaneous volatility of $\gamma L^{-1}$. The value $L$ at time $t$ proxies for macro and market information the market participants care about when trading the bonds. Such information is not already contained in the other state variables. A similar factor is used by Piazzesi (2005), who assumes an exogenous process to capture information not contained in the other state variables that could affect the yield curve. Given that investors value the safety and liquidity of US Treasuries, they are willing to forgo some interest in exchange for owning a high-liquid and safe debt instrument. Such interest is a significant time-varying convenience-yield component in Treasury yields, i.e., pushing down the yields, and is captured by $L$ in the model. A larger convenience yield reflects higher demand for Treasuries. As the instantaneous volatility of $L$ under the CEV process of Eq.(5) is a decreasing function of $L$, its volatility increases when $L$ is close to zero. This suggests that, when demand for Treasuries declines, future demand becomes more uncertain. Such a phenomenon is usually associated with high inflation risk (such as in the early 1990s), which reduces the demand for Treasuries, given the expectation of a drop in their prices under high inflation. On the other hand, a large $L$ suggests strong demand for Treasuries, lowering its volatility such that its dynamics become more deterministic under the drift term in Eq.(5), suggesting strong demand is persistent. Such an effect is similar to the leverage effect in equity markets in which stock price volatility increases as the stock price declines. This inverse relationship between the price and volatility is captured by the CEV process as shown by Cox (1975).

There are potentially three driving forces of the demand for Treasuries affecting $L$ after the GFC. First, the Fed conducted three rounds of quantitative easing, purchasing Treasuries and two rounds of

\[ dL_t = -\xi L_t dt + \gamma dZ_L \]  

(5)

7 The CEV model follows the following stochastic differential equation:

\[ dS_t = \mu S_t dt + \sigma S^{\beta/2} dZ_S \]

where $0 \leq \beta \leq 2$ is the elasticity parameter of the local volatility with the instantaneous volatility specified to be a power function of the underlying $S$. The model was introduced by Cox (1975) as an alternative to the geometric Brownian motion to model asset price. To keep the affine structure for the bond pricing model, $\beta$ must be either equal to zero or 1. As $L$ can be positive or negative, the square-root process with $\beta = 1$ is precluded and the only choice is to set $\beta$ equal to zero.

8 It is noted that the state variable $L$ is not related to the curvature of the yield curve.
operational twist in the market, which increased demand for Treasuries. Second, the increases in foreign reserves in emerging market economies due to net capital inflows enhanced demand. Third, risk aversion of investors increased demand for safe assets, including Treasuries, during the crisis period, in particular the European sovereign debt crisis. The model suggests that information related to these driving forces can be backed out from bond yield data, given that a solution for yields at time $t$ as a function of $X_t$. The contributions of these three driving forces are explored empirically in the next section.

Given Eqs.(3)-(5), the price of a zero-coupon bond with a maturity at time $\tau = T - t$ is given by:

$$P_t(\tau, r, \theta, L) = \mathbb{E}_t^Q \left[ \exp \left( - \int_t^T (r_t + L_t) dt \right) \right]$$

(6)

where the expectation is taken under the risk-neutral measure $Q$. We assume that the risk-neutral measure $Q$ has been chosen by the market in such a way that the adjusted discount rate $(r_t + L_t)$ is the effective interest rate matching the observed bond yields. The process $L$ is therefore introduced as a reduced form of a short-rate premium to capture the demand for Treasuries which carry a convenience yield. The assumption of an exogenous component in the discount factor has also been employed in the previous studies on term structures. Duffie and Singleton (1997) introduce an exogenous factor to capture the convenience yield in the pricing of interest rate swaps. Duffie and Singleton (1999) use it to capture the default component in the term-structure of corporate bonds. According to studies on convenience yield such as Krishnamurthy and Vissing-Jorgensen (2012), the observed Treasury interest rate is lower than the “true” risk-free interest rate by an amount of the convenience yield. The construction of the term structure model suggests that the short rate $r$ contains the information on the “true” risk-free interest rate. Strong demand for Treasuries will cause negative $L$, which pushes down the effective interest rate $(r_t + L_t)$. To preserve analytical tractability, we set the market price of risk as $(\lambda_r \sqrt{r}, \lambda_\theta \sqrt{\theta}, \lambda_L)$ for the state variables $(r, \theta, L)$ respectively. With the assumed functional form for risk premium, it is possible to rewrite Eqs.(3)-(5) under the risk-neutral measure $Q$, and the conditional expectation in Eq.(6) can be calculated by solving a partial differential equation:

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9 It is noteworthy that although the short rate is constrained to be non-negative, the effective interest rate in the bond pricing formula in Eq.(6) could be negative.
\[(r + L)P = \frac{\partial P}{\partial t} + \frac{1}{2} \sigma^2 r \frac{\partial^2 P}{\partial r^2} + (\kappa \theta - (\kappa + \lambda_r \sigma)) \frac{\partial P}{\partial \kappa} + \frac{1}{2} \eta^2 \theta \frac{\partial^2 P}{\partial \theta^2} + (\alpha \beta - (\alpha + \lambda_\theta \eta)) \frac{\partial P}{\partial \theta} + \frac{1}{2} \gamma^2 \frac{\partial^2 P}{\partial L^2} - (\xi L + \lambda_L Y) \frac{\partial P}{\partial L} \tag{7}\]

It follows from Duffie and Kan (1996) that Eq.(7) has the solution of the form:

\[P_t(\tau, r, \theta, L) = \exp[A(\tau) - B(\tau) r_t - C(\tau) \theta_t - D(\tau) L_t], \tag{8}\]

where the coefficient functions \(A(\tau), B(\tau), C(\tau), D(\tau)\) can be solved by a system of ordinary differential equations as:

\[\frac{dA(\tau)}{d\tau} = -\alpha \beta C(\tau) + \frac{1}{2} \gamma^2 D^2(\tau) + \lambda_L Y D(\tau) \tag{9}\]

\[\frac{dB(\tau)}{d\tau} = 1 - \frac{1}{2} \sigma^2 B^2(\tau) - (\kappa + \lambda_r \sigma) B(\tau) \tag{10}\]

\[\frac{dC(\tau)}{d\tau} = \kappa B(\tau) - \frac{1}{2} \eta^2 C^2(\tau) - (\alpha + \lambda_\theta \eta) C(\tau) \tag{11}\]

\[\frac{dD(\tau)}{d\tau} = 1 - \xi D(\tau) \tag{12}\]

for \(\tau \geq 0\) and \(A(0) = B(0) = C(0) = D(0) = 0\).

### 3. Model estimations

This section presents the empirical estimates of the term-structure model specified in Eqs.(3)-(5). We collect daily data of zero-coupon Treasury yields of constant maturities of 3-month, 6-month, and 1, 2, 3, 4, 5, 7, 10, 15, 20 and 30-year maturities for the sample period from January 1990 to December 2016.\(^{10}\) The weekly average of daily data is computed for the estimation, where the daily yields are stripped from the most recent auctioned-on-the-run US Treasury bills and bonds using standard bootstrapping. We use the closed-form maximum likelihood method developed by Ait-Sahalia and

\(^{10}\) All the data used for the model estimations are obtained from Bloomberg.
Kimmel (2010) to estimate the term structure model. As the cross-sectional number of observed bond yields is greater than the number of state variables, we follow previous studies to introduce measurement errors between the observed and model-implied yields. Specifically, we choose the 3-month, 10-year and 30-year maturities as the benchmark maturities (i.e., assuming no measurement errors) and use these bond yields to invert the state variables.

Table 1 reports the parameter estimates for the model. The $t$-ratios of the model parameters for the short rate $r$ are well above the conventional significance levels, indicating that the specification of the short rate under the CIR model is robust. The volatility for the long-term mean $\theta$ is significant, reflecting that the joint characterisation of the short- and long-term yield curve dynamics is adequate. The specification of the mean reversion for $\theta$ is, however, not significant. This is probably due to the substantial decline of long-term yields after the GFC. The significant estimations of the model parameters for the short-rate premium $L$ show the proposed CEV process adequately describes its dynamics, while the associated risk premium is not. The negative drift term $\xi$ suggests that $L$ is not stationary and the exogenous factors such as demand for Treasuries could push $L$ away from zero.

Table 2 reports the pricing errors for the non-benchmark maturities for the model. The results demonstrate that the model adequately fits for yields with all maturities, with the absolute pricing errors ranging from six to 29 bps.

Figure 3 graphs the path of the three state variables and federal funds rate. It shows that the short rate tracked closely to the federal funds rate before the 2008 GFC. When the federal funds rate was close to zero after 2008, the short rate also fell close to zero but with some deviations from 2009 to mid-2011. The gap between the short rate and federal funds rate increased after mid-2011 by around 0.5%. The gap remained even after the Fed raised the policy rate in December 2015. The short-rate premium $L$ was close to zero during most of the 1990s when the inflation risk was high.\textsuperscript{11} Under the CEV process in Eq.(5) for $L$ with small value, its volatility was high in that period, implying that demand for Treasuries was highly uncertain probably due to concern of further decline in their prices when inflation rose. $L$ dropped in the late 1990s and remained quite steady at the level of -40 bps until the 2008 GFC. In the model, a negative $L$ can be interpreted as stronger-than-usual demand for US

\textsuperscript{11} The exception was observed from 1992 to mid-1993 when it was positive, probably reflecting the fact that bond investors were seeking additional compensation for holding Treasury bonds amid a somewhat inflationary environment in the early 1990s.
Treasury bonds as safe assets, which could be partly due to the global savings glut since 2000. \( \frac{L}{L} \) edged downwards after the GFC, reflecting enhanced demand for Treasuries due to the quantitative easing policy and the purchases by emerging market economies for their increased foreign reserves due to net capital inflows. According to the property of the CEV dynamics, \( \frac{L}{L} \) becomes less volatile and more deterministic with more negative \( \frac{L}{L} \), suggesting that demand for Treasuries has been persistent since the GFC. The fall in \( \frac{L}{L} \) kept the effective rate \((r + L)\) close to the federal funds rate. Given the almost-zero Federal funds rate since 2008, the short-rate premium \( L \) was like a mirror image of the short-term \( r \), as shown in Figure 3.

Figure 3 shows that the estimated long-term mean level has persistently trended down, – a phenomenon that also occurs in long-term US Treasury yields. Although the current monetary policy stance undoubtedly has significant influence on the short-end of the yield curve, it is well known that the long-end yield curve contains expectations of future policy stances. The value of \( \theta \) edged up from 1\% to almost 2\% in late 2016 reflecting market expectation of the Fed’s plan to raise the policy rate in future.

4. Empirical analysis on short-rate premium and market anomalies

4.1 Dynamic relationship between short-rate premium and market anomalies

We now discuss the observations of the CIP deviations and negative swap spread possibly related to the short-rate premium \( L \). The CIP deviation is illustrated in Figure 4 as an average of the 1-year cross-currency swap basis spreads for the USD of the JPY, EUR and CHF, which have similar movements as shown in Figure 1. Market participants mainly use these three currencies to swap the foreign currencies into the USD, which share about one half of the total FX swap transactions.\(^{12}\) Panel A of Figure 4 demonstrates that the premium tracks quite closely with the CIP deviation during 2009-

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\(^{12}\) The daily average turnover of FX swaps of the JPY, EUR and CHF vis a vis the USD shared about 54\% of the total turnover in April 2016 as reported in the 2016 Triennial Central Bank Survey of foreign exchange and OTC derivatives markets conducted by the Bank for International Settlements.
2011. While there was a gap between them of about 20-30 bps during 2012-2016, their trends remain similar to each other. The premium shows a similar relationship with the negative swap spread. To illustrate any excessive adjustment of the short-rate premium after 2008, we take the average of $L$ during 1990-2008 which is about 21 bps and subtract this amount from $L$. The adjusted $L$ shown in Panel B of Figure 4 demonstrates that there has been excessive downwards adjustment of the premium since 2008, while the gaps between the adjusted $L$ and the two market anomalies were narrowed during 2012-2016.

We further study whether the dynamics of the short-rate premium $L$ and the two market anomalies are connected by using the cointegration method. The principal feature of cointegration is that a linear combination of non-stationary variables is stationary. This suggests that cointegrated variables move independently of each other but are linked by the stationary linear combination. This stationary relationship is regarded as a long-run equilibrium among the cointegrated variables. Under this equilibrium, a short-term deviation of a cointegrated variable from the others is expected to be temporary, and this cointegrated variable will gradually revert to the long-run relationship. Such relationship can be illustrated by considering a fall in $L$ (more negative short-rate premium) when it has an equilibrium relationship with the CIP deviation. This triggers a gap between the two variables. If this gap is large enough, the gap will ultimately be closed by either a larger CIP deviation or less negative $L$. The above illustration is a dynamical error-correction model. In the model, the short-term dynamics of the variables in the system are influenced by deviations from equilibrium. Assuming that the CIP deviation (swap spread) and the premium $L$ are “integrated of order 1” denoted by $I(1)$ (i.e., non-stationary in levels, but stationary in changes) and cointegrated, a model for the corresponding variables with the lagged changes of each variable can be expressed as:

$$
\Delta y_t = a_{y0} + \alpha_y (y_{t-1} - \beta L_{t-1}) + \sum_k b_{yk} \Delta y_{t-k} + \sum_k c_{lk} \Delta L_{t-k} + \epsilon_{yt}
$$

(13)

where $y_t$ is the CIP deviation or swap spread at time $t$, and $\alpha_y$ is greater than zero. As specified, the variable will change in response to stochastic shocks (represented by $\epsilon_{yt}$) and to the previous period’s gap from the long-run equilibrium (i.e., $(y_{t-1} - \beta L_{t-1}) \neq 0$). The long-run equilibrium is attained when $y_t = \beta L_t$. The parameter $\alpha_y$ is the speed of adjustment. In absolute terms, the larger $\alpha_y$ is, the greater the response of $y_t$ to the previous period’s gap from the long-run equilibrium. At the opposite extreme,
a very small value of $\alpha_y$ in absolute terms suggests that the CIP deviation (swap spread) is unresponsive to the last period’s equilibrium error. If $\alpha_y$ is equal to zero, the long-run equilibrium relationship does not appear and the model is not error-correction or cointegration. Thus, for a meaningful cointegration and error-correction model, the speed of adjustment $\alpha_y$ must be non-zero.

Table 3 provides summary statistics for the time series of the data in levels and changes. It also reports the Augmented Dickey–Fuller (ADF) and Phillip–Perrons (PP) test results. Using a maximum of four lags (a four-week period), both tests fail to reject at the 10% level the presence of a unit root for all levels of $L$, the CIP deviation and swap spread. However, both tests for the first differences are significant at the 5% level or less. Thus, their levels are non-stationary while the changes are stationary. This suggests that all pairs of $L$ and the CIP deviation (swap spread) are I(1) (the integrated of same order 1), which satisfies the requirement for the variables being cointegrated.

To test the cointegration between $L$ and the CIP deviation (swap spread), we use the Engle–Granger single-equation test which is proposed by Engle and Granger (1987) and is regarded as an easy and super-consistent method of estimation. It determines whether the residuals of the linear combination among the cointegrated variables estimated from the ordinary least squares method are stationary. Table 4 reports the cointegration tests between $L$ and the CIP deviation (swap spread). We employ the ADF and PP tests to check whether the residuals of the regression of $L$ on the CIP deviation (swap spread) are stationary. The critical values of the tests are based on MacKinnon (1996) and the lag length is determined by the Schwartz criterion. The results are significant for the CIP deviation and swap spread at the 5% and 1% levels respectively. Thus, we reject the null hypothesis that $L$ and the CIP deviation (swap spread) are not cointegrated in favour of the alternative hypothesis that there is at least one cointegrating vector.

Table 5 reports the estimated cointegrating vectors between $L$ and the CIP deviation (swap spread). The positive coefficients $\beta$ for the CIP deviation and swap spread are 0.354 and 0.199 respectively at the 1% level, suggesting that, other things being equal, a more negative $L$ would increase the CIP deviation (a more negative basis swap spread) and cause a more negative swap spread. As reported in Table 6, the estimates of the speed of adjustment $\alpha_y$ for the CIP deviation and swap spread are -0.026 and -0.027 respectively. They are negative but greater than -1, demonstrating that the CIP deviation and swap spread will subsequently adjust to restore the long-run equilibrium.
4.2 Empirical analysis on short-rate premium

We have shown that the short-rate premium in the Treasury yield curve is cointegrated with the CIP deviations and negative swap spreads. However, we have not directly linked this premium to the demand of Treasuries (for example, the convenience yield). To better understand the linkage and contemporaneous interactions between the short-rate premium and the factors related to the demand for Treasuries, we use a simple exploratory analysis to identify their relationships for the period of September 2008 to December 2016. It is noted that the regression analysis is based on intuitions rather than the basis of any theoretical model. As inspired by previous studies on demand for Treasuries including Longstaff (2004), we examine the following variables:

(i) Foreign Holdings: This variable is the change in the amount of Treasuries held by foreign investors. The short-rate premium should be more negative (i.e., an increase in the premium magnitude) when foreign holders increase their purchases of Treasury debt, particularly when there are increases in foreign reserves in emerging market economies due to net capital inflows after the GFC. The data on foreign holding of Treasuries is obtained from the Federal Reserve Board.

(ii) VIX: We use the CBOE VIX volatility index and the market volatility of the US S&P 500 index, to gauge the global risk appetite in the financial market. An increase in the VIX index is usually associated with heightened volatility across different asset classes in particular equities. It is a measure of investors’ aversion to volatility exposure and their willingness to put capital at risk. When investors become concerned about market risk, some may allocate their funds toward Treasuries as safe-haven assets. This suggests a negative relationship between the short-rate premium and the VIX index.\(^\text{13}\)

(iii) Treasury Buyback: This variable reflects the change in the amount of Treasury securities available to investors as a result of the Treasury buyback program. As the buyback could reduce the supply of Treasuries, we expect it has a negative relationship with the short-rate premium. However,\(^\text{13}\)

\(^\text{13}\) Alternatively, we used the percentage change in the amount of funds held in equity mutual funds to proxy risk appetite in the financial market. If investors are more likely to invest in equity mutual funds, they are less risk-averse and the demand for Treasury is expected to reduce. Thus, we might expect that there would be a positive relation between the short-term premium and the amount of funds flowing into equity mutual funds. The data on the amount of funds held in equity mutual funds are taken from the monthly releases of the Investment Company Institute. The regression result of using this variable rather than VIX, is statistically significant with the expected sign. The result is available upon request.
there were only three months in the estimation period to have Treasury buyback and their market values were very small.\textsuperscript{14}

(iv) \textit{QE1-3}: They are the one-zero dummy variables for the months of executing the three quantitative easing programs. As the policy increased demand for Treasuries, they are expected to have a negative relationship with the short-rate premium.

(v) \textit{OT1-2}: They are the one-zero dummy variables for the months of executing the two rounds of operational twist. As the operations increased the demand of the long-term Treasuries, they are expected to have a negative relationship with the short-rate premium.

(vi) \textit{BBB Bond Yield}: To control for the possibility of search-for-yield behaviour during the period of the ultra-accommodative monetary conditions, we include the Bloomberg 5-year Industrial BBB Bond Yield Index as an explanatory variable. If the search-for-yield activities were across the fixed-income markets (for example, investors buy both the relatively low-rated corporate bonds and Treasuries), the BBB bond yield is expected to have a positive relationship with the short-rate premium.

(vii) \textit{A Bond Yield}: Investors could buy fixed-income instruments with good credit ratings rather than Treasuries as safe assets. To control for such substitutional effect, the Bloomberg 5-year Industrial A Bond Yield Index is incorporated as an explanatory variable. Given that the decline in the yield index may reflect less demand for Treasuries, the A bond yield is expected to have a negative relationship with the short-rate premium.

(viii) $\Delta L_{t-1}$: To account for persistence, a lagged value of the short-rate premium is added as an additional explanatory variable in the regression.

Table 7 reports summary statistics for the explanatory variables used in the regression. To control for a possibly spurious relationship between the short-rate premium and one of the explanatory variables with similar time-series properties, the regression analysis is conducted by regressing the monthly figure for the foreign holding and buyback, and monthly average for the other explanatory variables on the first difference of the premium as follows:

\textsuperscript{14} In this program, the Treasury uses an auction format to repurchase longer-term Treasury bonds from the market.
\[ \Delta L_t = a_1 + a_2 \Delta L_{t-1} + a_3 \Delta Y_{BBB} + a_4 \Delta Y_{A} + a_5 \Delta \text{Foreign Holding}_{t-1} + a_6 \Delta \text{Buyback}_{t-1} + a_7 (\Delta \log(\text{VIX}))_{t-1} + a_8 \Delta E1_{t-1} + a_9 \Delta E2_{t-1} + a_{10} \Delta E3_{t-1} + a_{11} \Delta \text{OT1}_{t-1} + a_{12} \Delta \text{OT2}_{t-1} + \varepsilon_t \]  

The regression results are summarised in Table 8. The results indicate that there is a strong relationship between the short-rate premium and demand for Treasuries. Therefore, this exploratory analysis provides adequate support for viewing the short-rate premium containing information of a convenience yield of Treasuries. For instance, the relationship between the short-rate premium and the dummy variables of the quantitative easing and operational twist is almost always negative and significant (except insignificant QE3). This is consistent with the hypothesis that the ultra-accommodative monetary policy conducted after the GFC increased demand for Treasuries. The results also suggest that the relationship between the premium and changes in foreign holdings of Treasury debt is negative and significant. This supports the hypothesis that increases in emerging market economies’ foreign reserves due to net capital inflows after the GFC enhanced the demand for Treasuries. However, the Treasury buyback is not significant, probably due to very small amounts of buyback.

There is a significant negative relationship between the short-rate premium and the VIX index. Since the index reflects risk appetite in the financial market, this suggests that the premium becomes more negative during the periods when risk aversion increases. This is again consistent with the interpretation that the premium increases in magnitude when investors behave more cautiously. This relation provides empirical support for the hypothesis that the premium reflects the relative importance of Treasuries as safe assets for investors.

The lagged short-rate premium is significant, reflecting that there is a high degree of persistence in the premium. The change in the BBB bond yield is significant and positive in sign. This suggests that both the changes in the premium and the BBB bond yield reflects investors’ search-for-yield behaviour in a low interest rate environment. However, the relationship between the premium and the A bond yield is negative. The magnitude of the premium decreases (\( L \) is less negative) with a lower A
bond yield (higher A bond price). This indicates that there could be some substitutional effect of good-rating (A-rated) corporate bonds as safe assets, which reduces the demand for Treasuries.\textsuperscript{15}

This section shows that the short-rate premium embedded in the US Treasury yield curve tends to move in tandem with the CIP deviations and swap spread over time since 2009. The dynamics between the premium and the two market anomalies are found to be cointegrated, suggesting a long-run equilibrium between them. Given the empirical evidence of the relationship between the short-rate premium and demand for Treasuries, this suggests that the premium captures the information on Treasuries’ convenience yield. Based on the studies by Krishnamurthy and Vissing-Jorgensen (2012) and others, the “true” risk-free interest rate is higher than the observed short-term interest rate in the Treasury market (on which USD LIBOR is based). This manifests the measurement error of the risk-free interest rate because the convenience yield (or the short-rate premium in our estimation) is not taken into account. Both the FX swap and interest rate swap markets could have corrected this measurement error by adding spreads on LIBOR for pricing the corresponding instruments.

\textbf{5. Conclusion}

There have been two market anomalies, the widespread CIP deviations in FX swaps and the 30-year negative USD interest rate swap-Treasury spread, implying a risk-free arbitrage opportunity since the 2008 GFC. Using a three-factor non-Gaussian term structure model, the short-rate premium estimated from the US Treasury yield curve tends to move in tandem with the CIP deviations and the negative swap spread over time since 2009. The dynamics between this premium and the two market anomalies are found to be cointegrated, suggesting a long-run equilibrium between them.

Recent studies find Treasuries carrying a convenience yield due to demand for Treasuries. An important implication of the convenience yield is that the observed Treasury interest rate is lower than the “true” risk-free interest rate by an amount of the convenience yield. Our empirical analysis shows that the short-rate premium captures demand for Treasuries due to increased amounts of Treasuries.

\textsuperscript{15} For a robustness check, by using the spread between the BBB and A bond yields rather than their individual bond yields in Eq.(14), the spread is positively related to the premium at the 1\% significance level, consistent with the results based on the individual ratings’ bond yields.
held by foreign investors, the effects of the Fed’s quantitative easing policy and operation twist, and the risk aversion in the financial market. This indicates that the short-rate premium reflects the convenience yield embedded in the Treasury yield curve.

The above findings suggest that the anomalies manifest the measurement error of USD risk-free interest rates, consistent with recent studies common factor that the USD and its interest rates play an important role in determining the CIP deviations. Both the FX swap and interest rate swap markets could have corrected this measurement error by adding spreads on USD LIBOR, which makes reference to short-term Treasury interest rates. In other words, the FX swap and interest rate swap markets may have adjusted the USD risk-free interest rate in their corresponding instruments.
References


Figure 1. 30-year US dollar interest rate swap-Treasury spread and 1-year cross-currency basis swap spreads on US dollar LIBOR of Japanese yen (JPY), euro (EUR) and Swiss franc (CHF)
Figure 2. Taylor rule implied rate, 3-month US dollar LIBOR, 1-month Treasury yield and Fed funds target rate
Figure 3. Estimated state variables: short rate $r$, long-term mean $\mu$ and short-rate premium $L$; Federal funds rate; and rate implied by Taylor rule.
Figure 4. 30-year US dollar interest rate swap-Treasury spread, average 1-year cross-currency basis swap spreads on US dollar LIBOR of Japanese yen (JPY), euro (EUR) and Swiss franc (CHF), and short-rate premium $L$

Panel A

![Graph showing 30-year US dollar interest rate swap-Treasury spread, average 1-year cross-currency basis swap spreads on US dollar LIBOR of Japanese yen (JPY), euro (EUR) and Swiss franc (CHF), and short-rate premium $L$.]

Panel B

![Graph showing Adjusted short-rate premium $L$, 30-year USD swap spread, and average 1-year basis swap spread (JPY-EUR-CHF).]
Table 1. Maximum likelihood estimates

<table>
<thead>
<tr>
<th></th>
<th>Estimates</th>
<th>t-ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Short rate process (( \hat{\eta} ))</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean reversion (( \kappa ))</td>
<td>0.6973</td>
<td>22.21</td>
</tr>
<tr>
<td>volatility (( \sigma ))</td>
<td>0.0520</td>
<td>26.83</td>
</tr>
<tr>
<td>risk premium (( \lambda_r ))</td>
<td>-8.2475</td>
<td>-11.53</td>
</tr>
<tr>
<td><strong>Long-term mean process (( \hat{\theta} ))</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean reversion (( \alpha ))</td>
<td>0.0647</td>
<td>0.21</td>
</tr>
<tr>
<td>long-term mean (( \beta ))</td>
<td>0.0174</td>
<td>0.22</td>
</tr>
<tr>
<td>volatility (( \eta ))</td>
<td>0.0377</td>
<td>2.16</td>
</tr>
<tr>
<td>risk premium (( \lambda_\theta ))</td>
<td>0.8010</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Exogenous process (( \hat{L} ))</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>drift (( \xi ))</td>
<td>-0.0791</td>
<td>-4.79</td>
</tr>
<tr>
<td>volatility (( \gamma ))</td>
<td>0.0014</td>
<td>2.43</td>
</tr>
<tr>
<td>risk premium (( \lambda_L ))</td>
<td>-0.5826</td>
<td>-0.97</td>
</tr>
</tbody>
</table>

Note: The sample is weekly from January 1990 to December 2016.

Table 2. Mean and standard deviation of absolute pricing errors (in %)

<table>
<thead>
<tr>
<th></th>
<th>6-month</th>
<th>1-yr.</th>
<th>2-yr.</th>
<th>3-yr.</th>
<th>4-yr.</th>
<th>5-yr.</th>
<th>6-yr.</th>
<th>7-yr.</th>
<th>8-yr.</th>
<th>9-yr.</th>
<th>15-yr.</th>
<th>20-yr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.11</td>
<td>0.20</td>
<td>0.27</td>
<td>0.29</td>
<td>0.27</td>
<td>0.22</td>
<td>0.20</td>
<td>0.15</td>
<td>0.12</td>
<td>0.06</td>
<td>0.12</td>
<td>0.17</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.08</td>
<td>0.13</td>
<td>0.19</td>
<td>0.21</td>
<td>0.20</td>
<td>0.17</td>
<td>0.14</td>
<td>0.10</td>
<td>0.07</td>
<td>0.04</td>
<td>0.07</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Note: Absolute pricing errors are defined as absolute differences between the actual yields and the model-implied yields.
Table 3. Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Level</th>
<th>Change</th>
<th>Level</th>
<th>Change</th>
<th>Level</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.005</td>
<td>1.79E-06</td>
<td>-0.003</td>
<td>-4.12E-06</td>
<td>-0.002</td>
<td>-1.02E-05</td>
</tr>
<tr>
<td>Median</td>
<td>-0.005</td>
<td>0.000</td>
<td>-0.003</td>
<td>4.17E-07</td>
<td>-0.002</td>
<td>-5.00E-06</td>
</tr>
<tr>
<td>Maximum</td>
<td>-0.001</td>
<td>0.001</td>
<td>-0.001</td>
<td>0.002</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.008</td>
<td>-0.001</td>
<td>-0.007</td>
<td>-0.002</td>
<td>-0.006</td>
<td>-0.003</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.001</td>
<td>2.13E-04</td>
<td>0.001</td>
<td>3.37E-04</td>
<td>0.002</td>
<td>3.63E-04</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.138</td>
<td>0.048</td>
<td>-0.362</td>
<td>0.463</td>
<td>-0.368</td>
<td>-0.671</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.189</td>
<td>5.721</td>
<td>2.491</td>
<td>12.503</td>
<td>2.394</td>
<td>11.915</td>
</tr>
<tr>
<td>ADF test statistics</td>
<td>-1.679</td>
<td>-17.840 ***</td>
<td>-1.802</td>
<td>-27.013 ***</td>
<td>-2.249</td>
<td>-22.166 ***</td>
</tr>
<tr>
<td>Correlation with $L/\Delta L$</td>
<td>-</td>
<td>-</td>
<td>0.394</td>
<td>0.254</td>
<td>0.189</td>
<td>-0.269</td>
</tr>
<tr>
<td>Observations</td>
<td>419</td>
<td>418</td>
<td>418</td>
<td>416</td>
<td>405</td>
<td>391</td>
</tr>
</tbody>
</table>

Notes:
1. ***, ** and * indicate significance at levels of 1%, 5% and 10% respectively.
2. Both tests check the null hypothesis of unit root existence in the time series, assuming nonzero mean in the test equation.
3. The correlations for level of the variables are the correlations with $L$, and those for change are the correlation with $\Delta L$.

Table 4. Tests for cointegration

<table>
<thead>
<tr>
<th></th>
<th>Average 1-year basis swap spread (USD vs JPY-EUR-CHF)</th>
<th>30-Year USD interest rate swap rate-Treasury spread</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ADF test statistic</td>
<td>Phillips-Perron test statistic</td>
</tr>
<tr>
<td></td>
<td>-2.478 **</td>
<td>-3.034 ***</td>
</tr>
</tbody>
</table>

Notes:
1. ***, ** and * indicate significance at a level of 1%, 5% and 10% respectively.
2. The cointegration test uses the Augmented Dickey-Fuller and Phillips-Perron tests to check the null hypothesis that the residuals of the regression of $L$ on 1-year basis swap spread (USD vs JPY-EUR-CHF average) or 30-year USD interest rate swap rate-Treasury spread are non-stationary assuming zero mean in the test equation. The critical value of the test is obtained from MacKinnon (1996).
### Table 5. Estimates of cointegrating vectors (i.e., the long-run part of Eq.(13))

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>Average 1-year basis swap spread (USD vs JPY-EUR-CHF)</th>
<th>30-Year USD interest rate swap rate-Treasury spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$ ($\beta$)</td>
<td>0.354 ***</td>
<td>0.199 ***</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.001 ***</td>
<td>-0.001 ***</td>
</tr>
</tbody>
</table>

Notes: *** , ** and * indicate significance at a level of 1%, 5% and 10% respectively.

### Table 6. Estimation results of short-run dynamics

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>$\Delta$Average 1-year basis swap spread (USD vs JPY-EUR-CHF)</th>
<th>$\Delta$30-Year USD interest rate swap rate-Treasury spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-6.03E-06</td>
<td>-9.56E-07 *</td>
</tr>
<tr>
<td>Speed of adjustment</td>
<td>-0.026 *</td>
<td>-0.027 *</td>
</tr>
<tr>
<td>$\Delta L_{-1}$</td>
<td>0.019</td>
<td>-0.078</td>
</tr>
<tr>
<td>$\Delta L_{-2}$</td>
<td>-0.074</td>
<td>-0.075</td>
</tr>
<tr>
<td>Dependent variable$_{t-1}$</td>
<td>0.037</td>
<td>-0.296 ***</td>
</tr>
<tr>
<td>Dependent variable$_{t-2}$</td>
<td>0.018</td>
<td>-0.037</td>
</tr>
</tbody>
</table>

Notes: *** , ** and * indicate significance at a level of 1%, 5% and 10% respectively.
Table 7. Summary statistics for explanatory variables in regression of $L$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$BBB</td>
<td>-0.027</td>
<td>0.208</td>
<td>-0.640</td>
<td>-0.011</td>
<td>0.797</td>
<td>0.176</td>
</tr>
<tr>
<td>$\Delta$A</td>
<td>-0.023</td>
<td>0.200</td>
<td>-0.745</td>
<td>-0.003</td>
<td>0.831</td>
<td>0.188</td>
</tr>
<tr>
<td>$\Delta$Foreign holdings</td>
<td>33.152</td>
<td>53.391</td>
<td>-113.000</td>
<td>38.900</td>
<td>178.200</td>
<td>0.272</td>
</tr>
<tr>
<td>$\Delta$VIX</td>
<td>-0.005</td>
<td>0.181</td>
<td>-0.373</td>
<td>-0.018</td>
<td>0.705</td>
<td>0.071</td>
</tr>
<tr>
<td>Treasury Buyback</td>
<td>0.001</td>
<td>0.005</td>
<td>0.000</td>
<td>0.000</td>
<td>0.025</td>
<td>-0.042</td>
</tr>
</tbody>
</table>

Note: This table reports summary statistics for the explanatory variables in the regression of $\Delta L$. The variables $\Delta$BBB and $\Delta$A are the monthly changes in the Bloomberg 5-year US industrial BBB and A corporate bond yield indexes respectively measured in percentage points. $\Delta$Foreign Holdings is the monthly change in the total amount of foreign holdings of U.S. Treasury bonds measured in billions of dollars. $\Delta$VIX is the log-difference of monthly average of VIX. Treasury Buyback is the market value in $billions of all Treasury buybacks during the month. QE1, QE2 and QE3 are the dummy variables for the months of executing three quantitative easing programs. OT1 and OT2 are the dummy variables for the months of executing two rounds of operation twist. The data are monthly from September 2008 to December 2016. The number of observations for each time series is 100.

Table 8. Results from regression of $\Delta L$

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficients</th>
<th>$t$-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>7.61E-05*</td>
<td>1.677</td>
</tr>
<tr>
<td>$\Delta L_{t-1}$</td>
<td>0.273**</td>
<td>2.502</td>
</tr>
<tr>
<td>$\Delta$BBB$_t$</td>
<td>2.02E-03***</td>
<td>3.557</td>
</tr>
<tr>
<td>$\Delta$A$_t$</td>
<td>-2.24E-03***</td>
<td>-3.552</td>
</tr>
<tr>
<td>$\Delta$Foreign Holdings$_{t-1}$</td>
<td>-1.74E-06**</td>
<td>-2.459</td>
</tr>
<tr>
<td>$\Delta$VIX$_{t-1}$</td>
<td>-3.90E-04*</td>
<td>-1.676</td>
</tr>
<tr>
<td>Treasury Buyback$_{t-1}$</td>
<td>6.84E-03</td>
<td>0.919</td>
</tr>
<tr>
<td>QE1$_t$</td>
<td>-1.97E-03***</td>
<td>-9.445</td>
</tr>
<tr>
<td>QE2$_t$</td>
<td>-1.07E-03***</td>
<td>-5.899</td>
</tr>
<tr>
<td>QE3$_t$</td>
<td>6.40E-05</td>
<td>0.617</td>
</tr>
<tr>
<td>OT1$_t$</td>
<td>-1.09E-03***</td>
<td>-7.719</td>
</tr>
<tr>
<td>OT2$_t$</td>
<td>-4.00E-04***</td>
<td>-6.975</td>
</tr>
</tbody>
</table>

Adj. $R^2$ 0.368
No. of Observations 100

Note: The table presents the results of estimating $\Delta L$ on a monthly basis. ***, **, and * respectively indicate significance at the 1%, 5%, and 10% level. The robust t-statistics are based on White heteroskedasticity-consistent standard errors and covariance. Adjusted $R^2$ estimates are provided in the row labelled “Adj. $R^2$.$\Delta$BBB and $\Delta$A are the monthly changes in the Bloomberg 5 year US industrial BBB and A corporate bond yield indexes respectively in percentage points. $\Delta$Foreign Holdings is the monthly change in the total amount of foreign holdings of U.S. Treasury bonds measured in billions of dollars. $\Delta$VIX is the log-difference of monthly average of VIX. Treasury Buyback is the market value in $billions of all Treasury buybacks during the month. QE1, QE2 and QE3 are the dummy variables for the months of executing three quantitative easing programs. OT1 and OT2 are the dummy variables for the months of executing two rounds of operation twist. The sample is from September 2008 to December 2016.